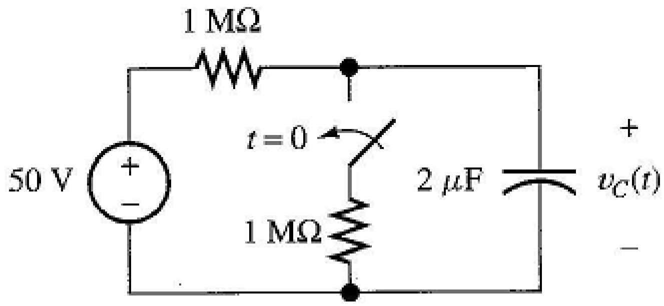
Q- Consider the circuit in the figure. The switch is closed for long time prior to t = 0.

(a) Determine the voltage V_c across the capacitor before t = 0 and long time after t = 0

(b) find the expression for the voltage across the capacitor as a function of time

(c) determine the time constant after the switch is closed

(d) plot the voltage across capacitor for -2 < t < 5 second.



Solution:

(a) If the switch is open long before t = 0 then at t = 0 the capacitor is fully charged to the potential difference 50 V, because as there is no current through the resistance, no drop in potential across the resistance. Hence V_C (t<0) = 50 V.

Long after the switch is closed a constant current is established in the two resistors and the charge on the capacitor becomes constant. The current in the two resistors is equal and hence as the two resistors having equal resistance the potential drop across each is same and is equal to 25 V. Hence the potential difference across capacitor is same as across second resistor and equal to V_C (t >>0) = 25 V.

(b) As soon as the switch is closed the current start growing in the two resistors and the capacitor starts getting discharge because initial current in the resistor is small. The potential difference across the capacitor is same as that across the second resistor. Let at time t the potential difference across the capacitor and the second resistor (both are in parallel) is V and hence charge on the capacitor is q = CV. If the current in the first resistor, is I then the current in the second is

$$I - (dq/dt) = I - C(dV/dt).$$ …Charge decreases, so dV/dt will be negative

Hence the potential difference across the capacitor = P.D. across R_2 will be

$$V = [I - C\,(dV/dt)]\,R$$ ………………… (1)

Now using Kirchhoff's loop rule for the loop containing the two resistances and the source we can write ε - IR - [I - C (dV/dt)] R = 0 { ε is e.m.f. of the source}

Or 2IR - CR (dV/dt) = ε ………… (2)

substituting the value of IR in equation (2) from equation (1)

2[V + CR(dV/dt)] - CR(dV/dt) = ε.

Or ε - 2V = CR(dv/dt)

Or $\frac{dV}{\varepsilon - 2V} = \frac{dt}{CR}$

Integrating with appropriate limits we get

$$\int_{\varepsilon}^{v} \frac{dV}{\varepsilon - 2V} = \int_{0}^{t} \frac{dt}{CR} \Rightarrow -\frac{1}{2}[ln(\varepsilon - 2V)]_{\varepsilon}^{V} = \frac{t}{CR} \Rightarrow ln\left[\frac{\varepsilon - 2V}{\varepsilon - 2\varepsilon}\right] = \frac{-2t}{CR}$$

Gives

$$\frac{\varepsilon - 2V}{-\varepsilon} = e^{-2t/CR}$$

$$\Rightarrow V = \frac{\varepsilon}{2}\left[1 + e^{-2t/CR}\right] = 25[1 + e^{-t}]Volts$$

This is the required potential difference as a function of time.

[We may have a cross check at t=0; we are getting v = ε, and at t = infinity v = ε/2 are in accordance with our discussion above.]

(c) The time constant is CR/2 = $(2*10^{-6}*10^{6})/2$ = 1 s.

(d) The qualitative graph is shown in the figure.