

Q- A piston-cylinder device contains N<sub>2</sub> gas, initially at 1.5 bar, 20<sup>0</sup>C, 0.05 m<sup>3</sup>. The gas is now compressed in a polytropic process, PV<sup>n</sup> = constant, to 4 bar, 150<sup>0</sup>C. Determine the polytropic exponent 'n', the work done on the gas and the heat transfer.

Nitrogen at given temperatures and pressure behaves as an ideal gas. An ideal gas obeys the gas equation PV = n RT under all processes. Thus we can write for the given gas

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Substituting given values we can calculate the volume of compressed gas as

[As we have pressure multiplied on both sides of equation, unit may not be changed]

$$\frac{1.5 \times 0.5}{(273+20)} = \frac{4 \times V_2}{(273+150)}$$

Gives  $V_2 = \frac{1.5 \times 0.5 \times (273+150)}{4 \times (273+20)} = 0.27 \text{ m}^3$  ----- (1)

As our process is given by

$$PV^n = \text{constant}$$

For the two states of the gas we can write

$$P_1 V_1^n = P_2 V_2^n$$

Gives  $1.5 \times (0.5)^n = 4 \times (0.27)^n$

Or  $1.5 \times 0.5^n = 4 \times 0.27^n$

Or  $\left(\frac{0.5}{0.27}\right)^n = \frac{4}{1.5}$

Or  $\ln\left(\frac{50}{27}\right)^n = \ln\left(\frac{8}{3}\right)$

Or  $n \times \ln(1.852) = \ln 2.667$

Gives  $n = \frac{\ln 2.667}{\ln 1.852} = 1.592$

Hence the exponent n = **1.592**

The work done by the gas in any process is given by

$$W = \int P \times dV$$

As the process is given by P V<sup>n</sup> = constant K (say)

Substituting we get the work done as

$$W = \int P \times dV = \int_{V_1}^{V_2} \left(\frac{K}{V^n}\right) dV$$

Or  $W = K \int_{V_1}^{V_2} V^{-n} dV = \frac{K}{-n+1} [V^{-n+1}]_{V_1}^{V_2}$

$$\text{Or } W = \frac{K}{-n+1} \left[ \frac{V}{V^n} \right]_{V_1}^{V_2} = \frac{K}{-n+1} \left[ \frac{V_2}{V_2^n} - \frac{V_1}{V_1^n} \right] = \frac{1}{-n+1} \left[ \frac{KV_2}{V_2^n} - \frac{KV_1}{V_1^n} \right]$$

But as  $K/V^n = P$  substituting for initial and final states we get

$$W = \frac{1}{-n+1} [P_2 V_2 - P_1 V_1]$$

Now here

$$P_1 = 1.5 \text{ bar} = 1.5 \times 10^5 \text{ Pa}$$

$$P_2 = 4.0 \text{ bar} = 4.0 \times 10^5 \text{ Pa}$$

$$V_1 = 0.5 \text{ m}^3$$

$$V_2 = 0.27 \text{ m}^3$$

And  $n = 1.592$

Substituting in above equation we get

$$W = \frac{1}{-1.592+1} [4.0 \times 10^5 \times 0.27 - 1.5 \times 10^5 \times 0.5]$$

$$\text{Or } W = \frac{1}{-1.592+1} [1.08 \times 10^5 - 0.75 \times 10^5]$$

$$\text{Or } W = -5.57 \times 10^4 \text{ J}$$

Hence the work done by the gas will be  **$W = -5.57 \times 10^4 \text{ J}$**

[Negative sign means that this work is done on the gas by external forces to compress it]

The heat transferred  $\Delta Q$  to the system is given by the first law of thermodynamics as

$$\Delta Q = \Delta U + W$$

Where  $\Delta U$  is increase in internal of the gas and  $W$  is work done by the gas.

As the increase in internal energy of the gas is given by  $(\nu C_v \Delta T)$ , we get

$$\Delta Q = \nu C_v \Delta T + W$$

[ $C_v$  is the molar specific heat and  $\nu$  is the number of moles in the gas]

$$\text{Now } \nu = \frac{PV}{RT} = \frac{1.5 \times 10^5 \times 0.5}{8.31 \times (273+20)} = 30.80 \text{ mol}$$

$$C_v = 20.8 \text{ J/(mol. K)}$$

Substituting all values we get the heat supplied to the gas

$$\Delta Q = \nu C_v \Delta T + W$$

$$\text{Or } \Delta Q = 30.80 \times 20.8 \times 130 + (-5.57 \times 10^4) = 8.33 \times 10^4 - 5.57 \times 10^4 \text{ J}$$

$$\text{Or } \Delta Q = 2.76 \times 10^4 \text{ J}$$

The data from Wikipedia

[http://en.wikipedia.org/wiki/Heat\\_capacity#Heat\\_capacity](http://en.wikipedia.org/wiki/Heat_capacity#Heat_capacity)