Suppose a current of I = 1 A flows through a conductor that is tapered. The initial diameter of the conductor is 1 mm with a length of 5 mm. The final diameter of the conductor is 3 mm with a length of 8 mm, the tapered section in between is 10 mm long. (a) What is the current density through all sections of this conductor? (b) Sketch the current density as a function of position throughout the conductor.

The diameter of the conductor is constant and 1mm from x = 0 to x = 5mm and then increases to 3mm up to x = 15mm. After that it is again constant up to x = 23 mm.

Current density is the current per unit area of cross section normal to the cross section. Hence the magnitude of current density in the first part from x = 0 to x = 5 mm, with constant cross section of diameter 1 mm, is given by

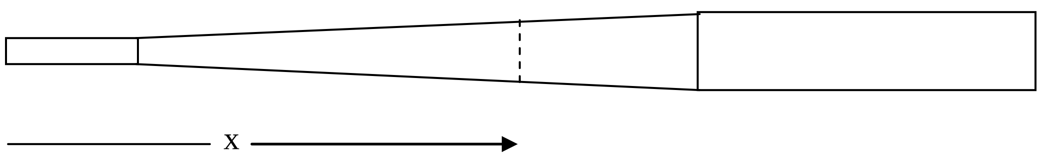
$$j = \frac{I}{A} = \frac{I}{\pi d^2/4} = \frac{4*1}{3.14*(1*10^{-3})^2} = 1.27*10^6 \text{ A/m}^2$$

The diameter of the wire is increasing in the next part from a = 1 mm to b = 3 mm hence the increase is of (b – a) in a length of *l* hence

Increase in the diameter per unit length is (b-a)/*l*

And hence the diameter at position x (5mm < x < 15 mm) is given by

$$d = 1 \text{ mm} + \left(\frac{b-a}{l}\right)(x-5) \text{ mm} = 1 \text{ mm} + 0.2\,(x-5) \text{ mm} = 0.2*x \text{ mm}$$



Hence the cross section area at position x mm is given by

$$A = \frac{\pi(0.2*x)^2}{4}$$

And hence the current density as a function of x is given by

$$j = \frac{I}{A} = \frac{I}{\pi d^2/4} = \frac{4*1}{3.14*(0.2*x*10^{-3})^2} = \frac{3.18*10^7}{x^2} A/m^2 \text{ (x is in mm)}$$

And for the last part as the diameter is constant the current density is given by

$$j = \frac{I}{A} = \frac{I}{\pi d^2/4} = \frac{4*1}{3.14*(3*10^{-3})^2} = 1.415*10^5 A/m^2$$

(b) The current density is constant at the ends but is inversely proportional to the distance in the middle and hence its slope is negative and increases in magnitude hence the graph can be qualitatively plotted as follow.