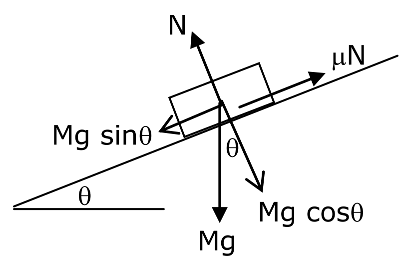
A block of mass M = 2.5 kg is released from rest and slides down an incline that makes an angle $\theta = 30^0$ with the horizontal. The coefficient of kinetic friction between the block and the incline is $\mu = 0.2$.

a) What is the acceleration of the block down the inclined plane?
b) How much work was done on the block by the Earth's gravitational force?
c) How much energy was expended in overcoming the frictional force (thus producing heat, etc.)?
d) What is the kinetic energy of the block?
e) The plane exerts a normal force (perpendicular to its surface) on the block. How much work was done on the block by this normal force?

The forces acting on the block are its weight Mg, the normal reaction N and the friction force μN. As we know that the block is moving along the incline and no motion in the direction normal to incline, we resolve forces in the direction along the incline and normal to incline. The component of the weight of the block along the incline is Mg sin θ and that normal to incline is Mg $\cos\theta$. The forces in the direction normal to the incline are balanced and hence we have



$Mg \cos\theta - N = 0$ gives $N = Mg \cos\theta$

And in the direction along the incline (down the incline positive)

$Mg \sin\theta - \mu N = Ma$ where a is the acceleration of the block along the incline

Substituting for N in equation 2 from equation 1 we get

$Mg \sin\theta - \mu\, Mg \cos\theta = Ma$

Or $a = g(\sin\theta - \mu\cos\theta) = 9.8*(\sin 30^0 - 0.2 \cos 30^0)$

$= 9.8(0.500 - 0.2*0.867) =$ **3.20 m/s/s**

b) How much work was done on the block by the Earth's gravitational force?

The product of gravitational force and the displacement in the direction of gravitational force give the work done by the gravity. As the the displacement along the incline is d the vertical displacement will be d*$\sin\theta$, and hence the work done by the gravitational force on the block is given by

$W_g = Mg*d*\sin\theta = 2.5*9.8*6*0.5 =$ **73.50 J**

c) How much energy was expended in overcoming the frictional force (thus producing heat, etc.)?

The product of friction force and the displacement in the direction of friction give the work by the friction. Hence

$W_f = -\mu N*d = -\mu(Mg\cos\theta)*d$

The negative sign is showing that the work is done against the friction and hence as the friction is non conservative force the work done will converted to the heat (etc) energy. So the heat energy produced is given by

$E = \mu$ (Mg $\cos\theta$)*d = 0.2*2.5*9.8*0.867*6 = **25.46 j**

d) What is the kinetic energy of the block?

The lost potential energy (work done by gravity) is used in two ways. According to work energy rule, the work done on a body is equal to the increase in its kinetic energy. Hence

Increase in KE = work done by gravity + work done by the friction

Or Final KE – initial KE = $W_g + W_f$

Or KE – 0 = Mg*d*$\sin\theta$ + (-) μ Mg $\cos\theta$*d)

Or KE = 73.50 – 25.46 = **48.04 J**

e) The plane exerts a normal force (perpendicular to its surface) on the block. How much work was done on the block by this normal force?

The work done by a force is given by

$W = \vec{F} \bullet d\vec{s}$ = F*ds*$\cos\theta$,

ds*$\cos\theta$ is the component of the displacement vector s in the direction of the force and hence we say that the work done is the product of the force and the displacement in the direction of the force.

As the displacement of the block is along the incline, the component of its displacement normal to the incline is zero i.e. the displacement in the direction of normal force is zero and hence the work done by the normal force is zero.

W_{norm} = **0**

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