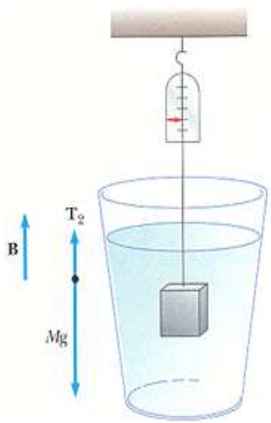
A 9.0 kg block of metal is suspended from a scale and immersed in water as in Figure P9.30. The dimensions of the block are 12.0 cm $\times$9.0 cm $\times$9.0 cm. The 12.0 cm dimension is vertical, and the top of the block is 5.00 cm below the surface of the water.

(a) What are the forces exerted by the water on the top and bottom of the block? (Do not ignore the effect of the air above the water. Take P_0 = 1.0130 $\times 10^5$ N/m^2.)
(b) What is the reading of the spring scale?
(c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.



A Reading

Buoyancy

In physics, buoyancy is an upward force on an object immersed in a fluid (i.e. a liquid or a gas), enabling it to float or at least to appear to become lighter. If the buoyancy exceeds the weight, then the object floats; if the weight exceeds the buoyancy, the object sinks. If the buoyancy equals the weight, the body has neutral buoyancy and may remain at its level. If its compressibility is less than that of the surrounding fluid, it is in stable equilibrium and will, indeed, remain at rest, but if its compressibility is greater, its equilibrium is unstable, and it will rise and expand on the slightest upward perturbation, but fall and compress on the slightest downward perturbation. It was the ancient Greek, Archimedes of Syracuse, who first discovered the law of buoyancy, sometimes called Archimedes' principle: *The buoyant force is equal to the weight of the displaced fluid*.
Typically, the weight of the displaced fluid is directly proportional to the volume of their displaced fluid (Specifically if the surrounding fluid is of uniform density.) Thus, among objects with equal masses, the one with greater volume has greater buoyancy.
Suppose a rock's weight is measured at 10 Newton when suspended by a string in a vacuum. Suppose that when the rock is lowered by the string into water, it displaces water whose weight is 3 Newton. The force it then exerts on the string from which it hangs will be 10 Newton minus the 3 Newton of buoyant force: 10 - 3 = 7 Newton.
Buoyancy is the underlying principle of many vehicles such as boats, ships, balloons, and airships.

Density

If the weight of an object is less than the weight of the fluid that the object would displace if it was fully submerged, then the object is less dense than the fluid and it floats at a level so it displaces the same weight of fluid as the weight of the object.

An object made of a material of higher density than the fluid, e.g. a metal object in water, can still float if it has a suitable shape (e.g. a hollow which is open upwards or downwards) that keeps a large enough volume of air below the surface level of the fluid. In that case, for the average density mentioned above, the air is included also, which may reduce this density to less than that of the fluid.

Solution

(a) What are the forces exerted by the water on the top and bottom of the block? (Do not ignore the effect of the air above the water. Take $P_0 = 1.0130 \times 10^5$ N/m^2.)

Force on the top is due to atmospheric pressure and the pressure due to water column above it. Area of the surface is 9cm*9cm = 81*10^{-4} m^2

As the force is pressure*area

$F_T = (P_0 + h_1 \rho g) A$ [here ρ is the density of water = 1000 kg/m^3]

= (1.0130*10^5 + 0.05*1000*9.8)*81*10^{-4}

= 824.499 N

And on the bottom

$F_B = (P_0 + h_2 \rho g) A$

= (1.0130*10^5 + 0.17*1000*9.8)*81*10^{-4}

= 834.0246 N

(b) What is the reading of the spring scale?

The scale will read apparent weight, which is (Weight of the block + force on top – force on bottom)

Or 9*9.8 + 824.499 - 834.0246 = 78.674 N

(c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.

The buoyant force = Weight of displaces water given by

V*ρ*g = (12.0*9.0*9.0)*10^{-6}*1000*9.8 = 9.5256 N

Difference = 834.0246 – 824.499 = 9.5256

Hence proved