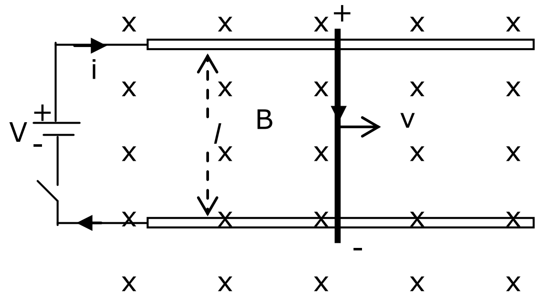
Q- An aluminum bar slides without friction along Horizontal metal rails through a vertical magnetic field B. The distance between the rails is *l*. The switch closes at t=0s, while the bar is at rest, and a battery of emf V, starts a current flowing around the loop. The battery has internal resistance r. The resistance of the rails and the bar are effectively zero.
a) Find the speed of the bar as a function of time
b) What is the terminal velocity of the bar (if the rails are too long)

Solution:

(a) Let at time the velocity of the rod be v then the magnitude of the motional EMF induced in the rod is given by



e = B*l*v

This induced EMF is produced in the rod due to its motion across the magnetic field, and its direction will be, according to Lenz law, such that it opposes the motion for that the direction of the induced current will opposite to that of the battery and hence the net EMF in the circuit will be

V - B*/*v

Thus the current in the loop formed by the battery, rails and the rod at time t is then given by

$$i = \frac{V - Blv}{r}$$

As the internal resistance of the battery r is the only resistance in the loop. Due to this current the force experienced by the rod is given by

$$\vec{F} = i\vec{l} \times \vec{B}$$

Whose direction is in the direction of motion of the rod and the magnitude will be

$$F = B * \left(\frac{V - Blv}{r}\right) * l$$

Now if m is the mass of the rod then according to Newton's second law

$$F = m\frac{dv}{dt} = B * \left(\frac{V - Blv}{r}\right) * l$$

Or $$\frac{dv}{V - Blv} = \frac{Bl}{mr}dt$$

Integrating the equation with proper limits we get the velocity of the rod as a function of time as

$$\int_0^v \frac{dv}{V - Blv} = \int_0^t \frac{Bl}{mr}dt$$

Or $$\frac{1}{-Bl}\left[\ln\left(V - Blv\right)\right]_0^v = \frac{Bl}{mr}t$$

Or $$\frac{1}{-Bl}\left[\ln\frac{(V-Blv)}{V}\right]=\frac{Bl}{mr}t$$

Or $$\ln\left(1-\frac{Blv}{V}\right)=-\frac{B^2l^2t}{mr}$$

Or $$1-\frac{Blv}{V}=e^{-Kt}$$ where $K=\frac{B^2l^2}{mr}$

Gives $$v=\frac{V}{Bl}\left(1-e^{-Kt}\right)$$

This is the equation for the velocity of the rod as a function of time.

(b)

The acceleration of the rod is given by

$$a=\frac{dv}{dt}=\frac{V}{Bl}\ (0+k\ e^{-Kt})$$

The equation shows that initially as t =0 the acceleration if maximum and decreases with time. After long time at t = ∞ it becomes zero. This means that after a large time the velocity of the rod becomes constant and this limiting value of velocity is called terminal velocity of the rod. This happens when the magnitude of induced EMF in the rod becomes equal to the EMF of the battery and in opposite to it so that net EMF and hence the current in the loop becomes zero and no force acts on the rod.

Hence the terminal velocity of the rod will be

$$v_T=\frac{V}{Bl}(1-0)=\frac{V}{Bl}$$

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