

Q1- During the time interval between $t = 0$ and $t = 17$ s, the total charge that crossed a plane across a wire at time t is given by $(16 \text{ C})(1 - e^{-t/5.8\text{s}})$, where t is in seconds. What is the maximum value of the current? What is the current at $t = 5.8$ s?

The current is the rate of flow of charge and hence if the total charge crossed is Q at time t and $Q + dQ$ at time $t + dt$ then the current at time t is given by

$$I = \frac{dQ}{dt} = \frac{d}{dt} [16(1 - e^{-t/5.8})]$$

Or $I = 16 \left[0 - e^{-t/5.8} \left(-\frac{1}{5.8} \right) \right]$

Or $I = 16 * e^{-t/5.8} \left(\frac{1}{5.8} \right)$

Or $I = 2.76 * e^{-t/5.8}$

Here the current is a function of time and decreasing continuously with it. The current will be maximum for $t = 0$ and minimum for $T = \text{infinity}$.

The maximum current is

$$I_{\text{max}} = 2.76 * e^0 = 2.76 \text{ A}$$

And the current at time 5.8 sec will be

$$I = 2.76 * e^{-5.8/5.8} = \frac{2.76}{e} = \frac{2.76}{2.72} = 1.015 \text{ A}$$

[This equation is the same as the equation for the charging of a capacitor and the behavior is similar to that. The charge is 0 at $t = 0$ and the charging current is maximum at $t = \text{infinity}$]

Q2- A tube of mercury with a resistivity of $9.84 \times 10^{-7} \Omega \cdot m$ has a 17.0 N/C electric field inside the column of mercury. How much current is flowing, if the diameter of the tube is 2.0 mm?

The current density is given by

$$j = E/\rho$$

Here E is electric field and ρ is resistivity of the conductor.

Or $j = \frac{17.0}{9.84 * 10^{-7}} = 1.73 * 10^7 \text{ A/m}^2$

Hence current through the column is

$$I = j * A = j * \pi d^2 / 4 = 1.73 * 10^7 * 3.14 * (2.0 * 10^{-3})^2 / 4 = 54.32 \text{ A.}$$