Q- A small block of mass m slides on a loop the loop track of radius r without friction. (a) Determine the minimum release height h so that the block completes the loop.

If the actual height of the release is 2h, calculate

(b) The normal force on the block by the track at the lowest point of the loop.
(c) The normal force on the block by the track at the top of the loop.

To move on a circular path a body requires a force towards the center of circle. This force is called centripetal force and is given by

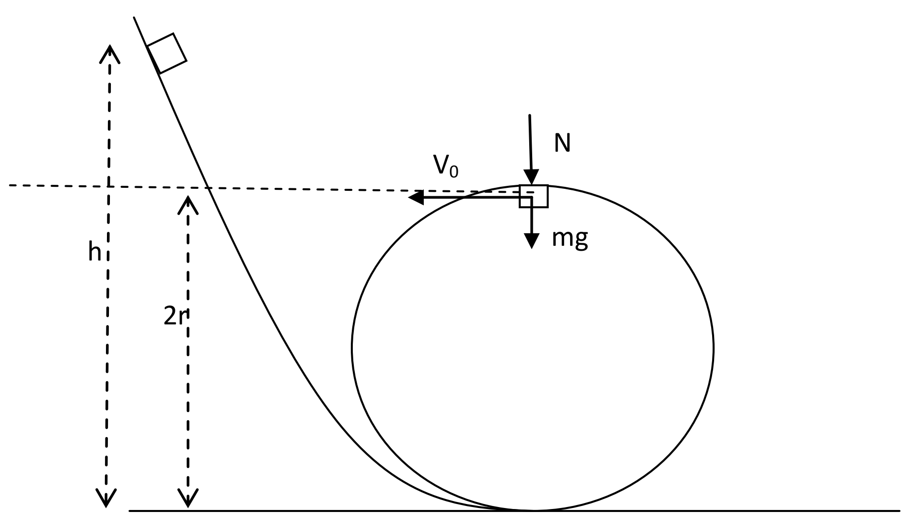
$$F_C = \frac{mv^2}{R}$$

On a vertical circular frictionless track the forces acting on the body are its weight and the normal reaction. The resultant of these two forces provides required centripetal force. The speed changes with the height and so will the required centripetal force, which decreases with the height. The normal reaction changes in such a way that the normal reaction and the component of weight along the center provide the necessary centripetal force. As mg is always downward, its maximum contribution to centripetal force is when it is at the top and at that point the normal reaction will be minimum. It will be just zero for the velocity of the block at the heightest point is given by

$$mg = \frac{mv_0^2}{r}$$

or $$v_0 = \sqrt{gr} \qquad \text{.....................(1)}$$

If the speed of the body at the top is less than this value the block will leave the circular track and if more than this normal reaction will provide the remaining force.



(a) Let if the block is released from minimum height h and it loops the loop. In this case the normal reaction at the top is just zero and its speed should be $v_0 = \sqrt{gr}$

As there is no non-conservative force acting on the block, its mechanical energy remains conserved and hence applying the law of conservation of mechanical energy at points of release and the top of the loop we get

Loss in potential energy = gain in kinetic energy

Or $\quad mg(h - 2r) = \frac{1}{2}mv_0^2$

Or $\quad mgh - 2mgr = \frac{1}{2}mgr$

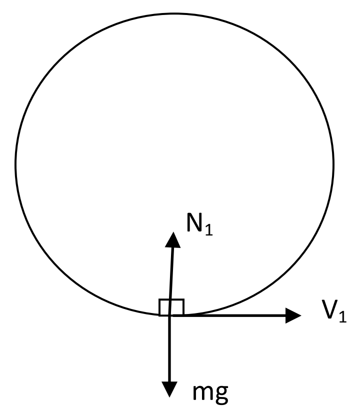
Or $\quad h = \frac{5}{2}\, r$

Hence to complete the loop the block must be released from a height more then

h = **5r/2**

(b) if the block is released from height 2h means 5r then the speed of the block at the bottom of the loop v_1 is again given by law of conservation of mechanical energy as

Loss in potential energy = gain in kinetic energy



Or $\quad mg\ 2h - 0\ = \frac{1}{2}mv_1^2$

Or $\quad v_1^2 = 4gh$

Or $\quad v_1 = \ 2\sqrt{gh}$

And in this case the normal reaction is given by

$$N_1 - \ mg = \frac{mv_1^2}{r}$$

Or $\quad N_1 = \ mg + m * \frac{4gh}{r}$

Or $\quad N_1 = \ \ mg\left(1 + \frac{4h}{r}\right)$

Or $\quad N_1 = \ \ mg\left(1 + \frac{4*5}{2}\right)$ [h/r = 5/2 from result of part a]

Or $\quad N_1 = \ \ 11\, mg$

Thus if the body is released from height 2h the normal reaction at the bottom will be **11mg** in upward direction.

(c) If the block is released from height 2h means 5r then the speed of the block at the top of the loop v_2 is given by law of conservation of mechanical energy as

Loss in potential energy = gain in kinetic energy

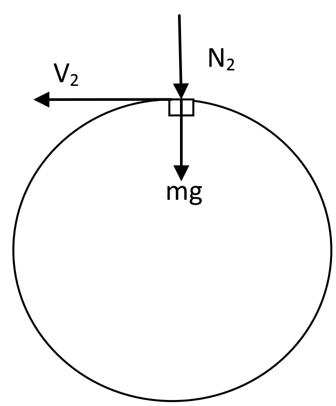
Or $$mg\ (2h-2r)=\frac{1}{2}mv_2^2$$

Or $$v_2^2=4gh-4gr$$

Or $$v_1=\ 2\sqrt{g(h-r)}$$

And in this case the normal reaction is given by

$$N_2+\ mg=\frac{mv_2^2}{r}$$



Or $$N_2=-\ mg+\frac{m}{r}\ (4gh-4gr)$$

Or $$N_2=\frac{4mgh}{r}-\ 5\ mg$$

Or $$N_2=mg\left(\frac{4h}{r}-\ 5\right)$$

Or $$N_2=mg\left(\frac{4*5}{2}-\ 5\right)$$

Or $$N_2=5\ mg$$

Thus if the body is released from height 2h the normal reaction at the top will be **5mg** in downward direction.