

Two point charges  $Q_1$  and  $Q_2$  are placed at a distance  $d$  apart.

(a) What is the force acting on each of the charge?

(b) Derive an expression for electric potential at any point in the space in terms of its coordinates. Consider potential to be zero at infinity. Consider  $Q_1$  is at origin and  $Q_2$  at  $(d, 0, 0)$ .

(a) Coulombs law gives the force between two point charges and the magnitude of the force is given by

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}$$

Here  $q_1$  and  $q_2$  are the charges;  $r$  is the distance between them,  $\epsilon_0$  is the permittivity of free space and  $\hat{r}$  is the unit vector in the direction of line joining the charges.

Thus the force on the charge  $Q_2$  on the right, due to charge  $Q_1$  on the left is given by

$$\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\epsilon_0 d^2} \hat{i}$$

Thus the magnitude of the force on  $Q_2$  is  $\frac{Q_1 Q_2}{4\pi\epsilon_0 d^2}$  and it is to the right or positive  $x$  direction.

Now according to the Newton's third law of motion, every action has equal and opposite reaction, the equal force is experience by  $Q_1$  due to  $Q_2$  and this is given by

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 d^2} (-\hat{i})$$

Thus the magnitude of the force experienced by  $Q_1$  on the left due to  $Q_2$  is the same  $\frac{Q_1 Q_2}{4\pi\epsilon_0 d^2}$  but its direction is to the left or negative  $x$  direction.

(b) The electric potential at a point in an electric field is the electrostatic potential energy stored per unit (test) charge at that point.

If a charge  $q$  posses electrostatic potential energy  $U$  at a point then potential at that point is given by

$$V = U/q$$

Potential at a distance  $r$  from a point charge  $Q$  is given by ( $V = 0$  at infinity)

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

As the energy and hence the potential is a scalar quantity, the potential at a point due to number of point charges is the scalar sum of the potential due to different charges.

Let  $\vec{r}_1$  is the position vector of point  $P(x, y, z)$ . The distance of  $P$  from the origin or from charge  $Q_1$  will be then given by

$$r_1 = \sqrt{x^2 + y^2 + z^2}$$

And thus the potential due to charge  $Q_1$  at  $P$  is given by

$$V_1 = \frac{Q_1}{4\pi\epsilon_0 r_1} = \frac{Q_1}{4\pi\epsilon_0 \sqrt{x^2 + y^2 + z^2}} \text{----- (1)}$$

Now the distance of point  $P$  from the second charge  $Q_2$  is given by

$$r_2 = \sqrt{(x - d)^2 + y^2 + z^2}$$

And thus the potential due to charge  $Q_2$  at  $P$  is given by

$$V_2 = \frac{Q_2}{4\pi\epsilon_0 r_2} = \frac{Q_2}{4\pi\epsilon_0 \sqrt{(x - d)^2 + y^2 + z^2}} \text{----- (2)}$$

Hence the potential at point  $P$  due to both charges is given by

$$V = V_1 + V_2$$

Substituting values from equations (1) and (2) we get

$$V = \frac{Q_1}{4\pi\epsilon_0 \sqrt{x^2 + y^2 + z^2}} + \frac{Q_2}{4\pi\epsilon_0 \sqrt{(x - d)^2 + y^2 + z^2}}$$

Or 
$$V = \frac{1}{4\pi\epsilon_0} \left( \frac{Q_1}{\sqrt{x^2 + y^2 + z^2}} + \frac{Q_2}{\sqrt{(x - d)^2 + y^2 + z^2}} \right)$$

With the help of this expression we can calculate the electric potential at any point  $P(x, y, z)$