Q- The current density in a cylindrical wire varies with the distance from the axis of the wire r, as j = j_0 (r/a). Hear j_0 is a constant and 'a' is the radius of the wire. Find
(a) Magnetic field at the axis of the wire
(b) Magnetic field at distance r from the axis of the wire.
(c) Magnetic field at the surface of the wire
(d) Current in the wire

The ampere's circuital law gives the magnitude of the magnetic field for a symmetric current distribution and is

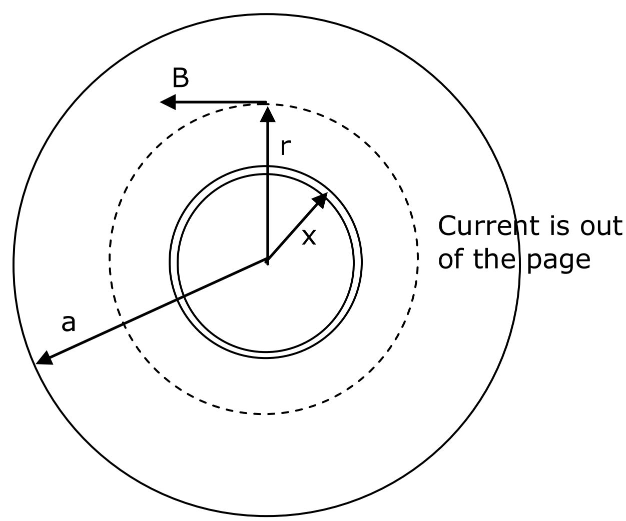
$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

Here the first term gives the line integration of the magnetic field for a closed loop and the I is the current within the loop.

Inside the wire as the current is distributed uniformly about its axis the field will be radially symmetric in magnitude and the direction will be tangential. The magnetic lines of force will be circular with the center at the axis of the wire. Hence considering a circular loop, with axis of the wire as center, the magnitude B of the magnetic field will be constant and can be taken out of the line integral. Then the rest of the integral will be the summation of the length of the closed loop and will be equal to 2πr. Thus for a circular loop of radius r

$$\oint \vec{B}.d\vec{l} = \oint B\, dl \cos\theta = B \oint dl = B * 2\pi r$$

Now to calculate the current within the loop consider a thin ring between the radii x and x + dx with the center at the axis.



Area of this ring will be dA = 2πx*dx and hence the current in this infinitesimally thin ring will be

dI = J*dA = $J_0 * \frac{x}{a} * 2\pi x * dx$

Hence total current within the loop of radius r will be

I = $\int dI = \frac{2\pi J_0}{a} \int_0^r x^2 * dx = \frac{2\pi J_0 r^3}{3a}$

Hence substituting the values in equation of ampere's law we have

$$B * 2\pi r = \frac{\mu_0 2\pi J_0 r^3}{3a}$$

This gives B as a function of r

$$B = \frac{\mu_0 J_0 r^2}{3a}$$

Now consider the different cases.

(a) For r = 0, B = 0. We have to consider the axis of the wire as loop and hence the area for the loop is zero giving the current in the loop zero and hence the magnetic field will be zero.

(b) Substituting r = a/2 and the other quantities we have

$$B = \frac{\mu_0 J_0 r^2}{3a} = \frac{\mu_0 J_0 (a/2)^2}{3a} = \frac{\mu_0 J_0 a}{12}$$

(c) Substituting r =a we have

$$B = \frac{\mu_0 J_0 r^2}{3a} = \frac{\mu_0 J_0 a^2}{3a} = \frac{\mu_0 J_0 a}{3} =$$

(d) For the total current through the wire we can integrate the current through the thin ring for the radius of wire as

$$\mathrm{I} = \int dI = \frac{2\pi J_0}{a}\int_0^a x^2 * dx = \frac{2\pi J_0 a^3}{3a} = \frac{2\pi J_0 a^2}{3}$$