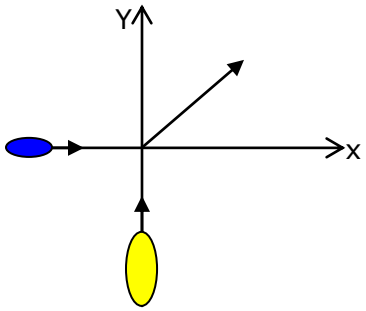
Q- A car with a mass of m_{car} = 2080 kg moving towards east and a truck with mass m_{truck} = 8990 kg moving towards north with velocity v_{truck} = 8 m/s, enter an intersection. When the two collide they stick together and slide at an angle of $\theta = 30^0$ north of east.
a) What is the speed of the pair after the collision?
b) What is the eastward component of the momentum of the pair after the collision?
c) At what speed was the car traveling before the collision?
d) What is the kinetic energy of the pair before the collision?
e) What is the kinetic energy of the pair after the collision?

Answer:
Considering the x and y axis conventionally as in figure and i and j the unit velocity vectors in x and y directions respectively. Let the speed of the car be v_x. The initial velocities of the car and the truck are given in vector form by

$$\vec{v}_{car} = v_x\hat{i} \qquad \text{and} \qquad \vec{v}_{truck} = 8\hat{j}$$



As there no external horizontal force on the system of the car and the truck, and as the car and the truck stick together, the combined mass will move such that the total momentum of the system conserved. Hence if the combined mass moves with speed v after collision then we have

$$m_c * \vec{v}_{car} + m_t * \vec{v}_{truck} = (m_c + m_t)\vec{v}$$

or $$2080 * v_x\hat{i} + 8990*8\ \hat{j} = (2080 + 8990)\ \vec{v}$$

or $$\vec{v} = \frac{2080 * v_x}{11070}\hat{i} + \frac{71920}{11070}\hat{j} = 0.188 * v_x\hat{i} + 6.497\hat{j}$$

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a) What is the speed of the pair after the collision?

The direction of the velocity of the combined mass after collision is given by

$$\tan\theta = 6.497/(0.188*v_x) = \tan 30^0 = 0.5774$$

gives v_x = 59.86 m/s

Hence the eastward component of the speed of the combined mass is given by

$$0.188*v_x = 11.25 \text{ m/s}$$

so the speed of the combined mass is given by

$$v = \sqrt{11.25^2 + 6.497^2} = 13.0 \text{ m/s}$$

v_{pair} = 13.0 m/s

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b) What is the eastward component of the momentum of the pair after the collision?

As there is no external force on the system, the eastward component of momentum remains conserved and hence it is the initial momentum of the car which is

$2080*59.86 = 1.245*10^5$ kg.m/s

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c) At what speed was the car traveling before the collision?
The speed of the car before collision is already calculated

Vx = 59.86 m/s

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d) What is the kinetic energy of the pair before the collision?

The kinetic energy of the pair before collision is the sum of their kinetic energy

$K= ½\ mc*v_x^2 + ½\ mt*8^2 = 0.5*2080*59.86^2 + 0.5*8990*64 = 4.014*10^6$ J

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e) What is the kinetic energy of the pair after the collision?

The kinetic energy after collision after collision is

$K' = ½\ (mc + mt)*v^2 = 0.5*11070*13^2 = 9.354*10^5$ J