Q- A circuit contains a loop with an inductor and a capacitor which and produces an oscillating current. In this *LC* circuit, $L$ = 3.5 mH and $C$ = 2.6 µF. At t = 0 the charge on the capacitor is zero and the current in the circuit is 1 A.

(a)What is the maximum charge $Q$ that will appear on the capacitor?

(b)What is the total energy $U$ oscillating in the system?

(c)At what fraction of the period of oscillation $T$ will the energy stored in the capacitor be **increasing** at its greatest rate?

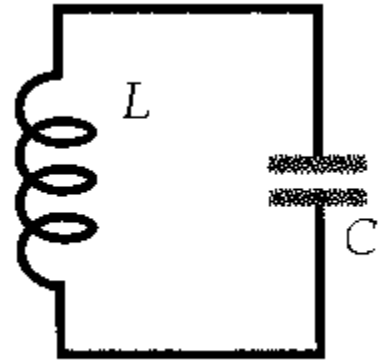(d) What is this greatest rate at which energy is transferred to the capacitor?

**Reading:**

Let the charge on the capacitor is q at time t then as there is no source the net potential drop in the circuit is zero or the induced EMF in the inductor will be dropped across the capacitor and hence we can write

$$\left(-L\frac{dI}{dt}\right) = \frac{q}{C}$$

Or $$\left(L\frac{d^2q}{dt^2}\right) = -\frac{q}{C}$$

Or $$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

The equation is similar to the equation for simple harmonic oscillator

$$\frac{d^2x}{dt^2} = -Kx$$

And hence the solution is given by

$$q = Q\sin\left(\sqrt{\frac{1}{LC}}\,t\right)$$

This gives the charge on the capacitor as a function of time.

The current in the circuit is given by

$$I = \frac{dq}{dt} = \frac{Q}{\sqrt{LC}}\cos\left(\sqrt{\frac{1}{CL}}\,t\right)$$

The energy stored in the capacitor at time t will be

$$U_C = \frac{q^2}{2C} = \frac{Q^2}{2C}\sin^2\left(\sqrt{\frac{1}{LC}}\,t\right)$$

**Answer**

(a)What is the maximum charge $Q$ that will appear on the capacitor?

The energy stored in the inductor is given by

$$U = \tfrac{1}{2} LI^2$$

When this whole energy is transferred to the capacitor and the current becomes zero then the charge on the capacitor is maximum and hence if the maximum charge is Q then

$$Q^2/(2C) = \tfrac{1}{2} LI^2$$

Or  $Q^2 = CLI^2 = 2.6*10^{-6}*3.5*10^{-3}*1^2 = 9.1*10^{-9}$

Or  $Q = 9.54*10^{-5}$ C = **95.4 μC**

(b)What is the total energy $U$ oscillating in the system?

The total energy of the system is given by

$$U = \tfrac{1}{2} LI^2 = 0.5*3.5*10^{-3}*(1)^2 = 1.75*10^{-3} \text{ J} = \textbf{1.75 mJ}$$

(c)At what fraction of the period of oscillation $T$ will the energy stored in the capacitor be **increasing** at its greatest rate?

The energy stored in the capacitor is given by $q^2/(2C)$ and hence it will increase at the greatest rate when the charge on the capacitor is increase at greatest rate. The charging current in the capacitor is passing through the inductor hence the charge on the capacitor will increase at maximum rate when the current through the inductor is maximum or at the initial moment. Hence the energy in the capacitor is increases by the greatest rate at t = 0 and hence the fraction t/T = 0.

(d) What is this greatest rate at which energy is transferred to the capacitor?

The energy stored in the capacitor at time t will be

$$U_C = \frac{q^2}{2C} = \frac{Q^2}{2C}\sin^2\left(\sqrt{\frac{1}{LC}}\right)t$$

Hence the rate at which the energy is transferred will be given by

$$r = \frac{dU_C}{dt} = \frac{Q^2}{2C}\left[2\sin\left(\sqrt{\frac{1}{LC}}\right)t * \cos\left(\sqrt{\frac{1}{LC}}\right)t * \left(\sqrt{\frac{1}{LC}}\right)\right]$$

Or  $$r = \frac{Q^2}{2C} * \sqrt{\frac{1}{LC}}\left[\sin 2\left(\sqrt{\frac{1}{LC}}\right)t\right]$$

Greatest value for sine function is 1 hence

$$r_{max} = \frac{Q^2}{2C} * \sqrt{\frac{1}{LC}} = \frac{1.75*10^{-3}}{\sqrt{3.5*10^{-3}*2.6*10^{-6}}} = 18.345W$$