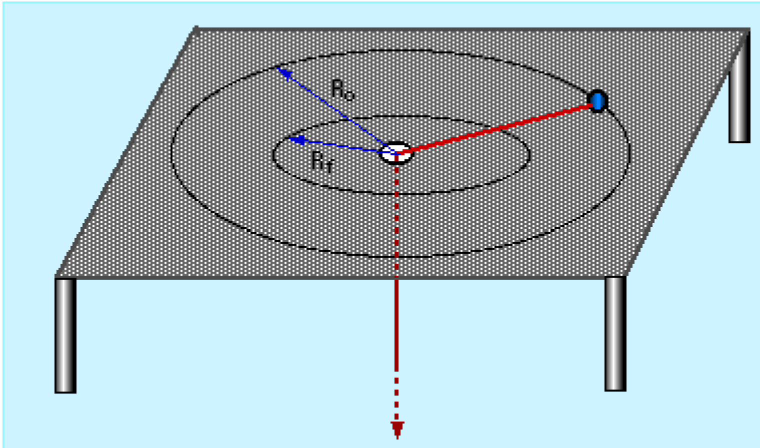
Q- A small block of mass 0.91 kg slides without friction on a horizontal table. Initially it moves in a circle of radius r_0 = 0.63 m with a speed 1.5 m/s. It is held in its path by a string that passes through a small hole at the center of the circle. The string is then pulled down a distance of $r_0 - r_1$ = 0.12 m, leaving it at a radius of r_1 = 0.51 m. It is pulled so slowly that the object continues to move in a circle of continually decreasing radius. How much work was done by the force to change the radius from 0.63 m to 0.51 m?

As there is no friction, the work done by the force is equal to change in the kinetic energy of the block.



As the force acting on the block is towards the center of the circular path, the net torque on the block about the center of the rotation is zero. $\left(\vec{\tau} = \vec{r} \times \vec{F}\right)$ (r and F are in the same direction)

With no torque on the rotating block the angular momentum of the block remains conserved. Hence if the speed of the block when its radius is r_1 is v then equating the final and initial angular momentum of the block about the center of the circle we have

$m*v*r_1 = m*v_0*r_0$ {the angular moment is the moment of momentum}

or $v = v_0*r_0/r_1$

or $v = 1.5*0.63/0.51 = 1.853$ m/s

Hence the increase in kinetic energy i.e. the work done on the block

$W = ½ m (v^2 – v_0^2) = 0.5*0.91(1.853^2 – 1.5^2) = 0.538$ J

Answer: W = 0.538 Joules

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