

Q- A block of mass 5 kg hangs on a spring. When a second block with an identical mass of 5 kg is tied to the first, the spring stretches an additional $h_0 = 1$ m.

a) What is the value of the spring constant k ?

Now the string is burned and the second block falls off.

b) How far above its original position does the remaining block attain its maximum speed?

c) What is the maximum speed attained by the remaining block?

Answer

a) The elongation (extension) in an ideal spring is proportional to the tension in the spring and hence the spring constant is the ratio of the force to the extension or the force per unit extension. Here the additional force $mg = 5 \times 9.8$ N is stretching the spring by 1m, hence the force constant of the spring is given by

$$K = 5 \times 9.8 / 1 = 49 \text{ N/m}$$

Now the string is burned and the second block falls off.

b) The speed of the remaining block will be maximum when the acceleration of the block or the net force on it is zero i.e. its first equilibrium position, which is 1m above the second equilibrium position. Thus the gain in height to the first block after the string is burned to reach maximum speed will be $h = 1.0$ m

c) What is the maximum speed attained by the remaining block?

The kinetic energy gained is equal to the difference of the loss in elastic potential energy and gain in gravitational potential energy.

Now at first the spring was stretched by $x_1 = 1$ m due to the first block and after the second block is attached the total extension becomes $x_2 = 2$ m and hence the loss in the elastic potential energy of the spring

$$\Delta E_s = \frac{1}{2} Kx_2^2 - \frac{1}{2} Kx_1^2 = 0.5 \times 49 \times (4 - 1) = 73.5 \text{ J}$$

The gain in the gravitational potential energy of the block is

$$\Delta U = m \times g \times h = 5 \times 9.8 \times 1 = 49 \text{ J}$$

Hence the maximum kinetic energy $= \frac{1}{2} mv^2 = \Delta E_s - mgh = 73.5 - 49 = 24.5 \text{ J}$

Gives the maximum speed of the block $v = \sqrt{\frac{24.5 \times 2}{5}} = 3.13 \text{ m/s}$
