Q- A long conducting pipe of radius R carries a uniformly distributed current I_1 along its length. A long conducting wire carries a current I_2 is placed at a distance of 3R from the axis of the pipe and parallel to it. Find
(a) Magnetic field at the center of the pipe
(b) Magnetic field at a point distance 2R from the axis of the pipe towards wire.

(a) Let the current in the wire I_1 and in the pipe I_2 are into the page. The magnitude of the magnetic field at the center of the pipe due to current I_2 in wire will be then

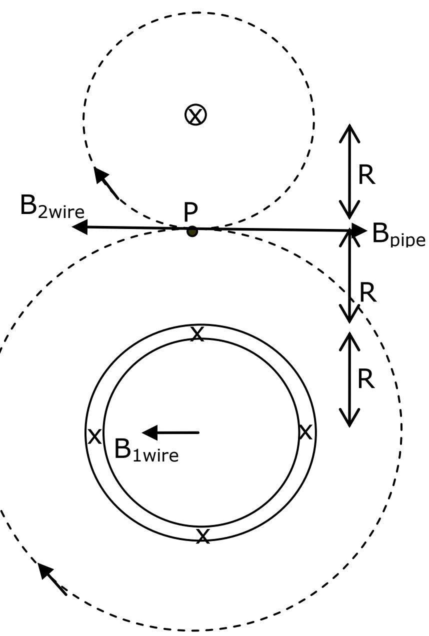
$$B_{1wire} = \frac{\mu_0 I_2}{2\pi(3R)} = \frac{\mu_0 I_2}{6\pi R}$$

And according to right hand rule the direction of this field will be to the left.

We know by ampere's rule that the magnetic field inside a pipe having uniformly distributed current is zero. It means that the magnetic field at the axis of the pipe is only due to current carrying wire and is towards left and hence net field at the axis of the pipe is

$$B_1 = \frac{\mu_0 I_2}{6\pi R}$$

And the direction is to the left.



(b) At point P at a distance R from the wire magnetic field due to wire will be

$$B_{2wire} = \frac{\mu_0 I_2}{2\pi R} \quad \text{(To the left)}$$

And the field at point P (outside) due to the current in the pipe will be same as that of a wire at the axis and hence

$$B_{pipe} = \frac{\mu_0 I_1}{2\pi * 2R}$$

Or $$B_{pipe} = \frac{\mu_0 I_1}{4\pi R}$$

The direction of this field will be to the right.

Thus the resultant field at P is given by

$$B = B_{2wire} - B_{pipe}$$

Or $$B = \frac{\mu_0 I_2}{2\pi R} - \frac{\mu_0 I_1}{4\pi R}$$

Or $$B = \frac{\mu_0}{4\pi R}\left(2I_2 - I_1\right)$$