Q1- Neutron stars are extremely dense objects with a mass comparable to the mass of the sun but a radius of only several thousand meters. Consider a neutron star of mass M = $1.99*10^{30}$ kg and a radius of R = 10.2 km.

(a) What is the gravitational acceleration near the surface of the star?
(b) If an incredibly heat resistant object was dropped near the surface of the star, how fast would it be going after it had fallen a distance of h = 1.5 m?
(c) Assuming uniform density, how much would 1.2 cubic centimeters of neutron star material weigh on the surface of the earth?

(a) The gravitational acceleration at the surface of a spherical body is given by

$$g = - GM/R^2 = - 6.67*10^{-11}*1.99*10^{30}/(10.2*10^3)^2 = - 1.276*10^{12}\ m/s^2.$$

Or $g = - 1.276*10^{12}\ m/s^2$

(b) If an incredibly heat resistant object was dropped near the surface of the star, how fast would it be going after it had fallen a distance of h = 1.5 m?

The velocity of the object is given by the equation

$[v^2 = u^2 + 2as]$

Or $v^2 = 0 + 2(1.276*10^{12})*1.5 = 3.827*10^{12}$

Gives $v = 1956365.12$ m/s
Or $v = 1.956*10^6$ m/s

(c) Assuming uniform density, how much would 1.2 cubic centimeters of neutron star material weigh on the surface of the earth?

The mass of the star $M = 1.99*10^{30}$ kg

The volume of the star $V = (4/3)\ \pi R^3 = (4/3)*3.14*(10200)^3 = 4.443*10^{12}\ m^3$.

Hence the density of the material of the star is given by

$$d = M/V = 1.99*10^{30}/(4.443*10^{12}) = 4.479*10^{17}\ kg/m^3.$$

And mass of 1.2 cubic cm = $1.2*10^{-6}\ m^3$ volume is

Or $m = 4.479*10^{17}*1.2*10^{-6} = 5.375*10^{11}$ kg

And its weight

$$W = mg = 5.375*10^{11}*9.8 = 5.267*10^{12}\ N$$

Q2- Two masses m_1 and m_2 ($m_1+m_2 = m$) are separated by a distance r_0. On releasing under the force of gravity they start moving from rest. Find their velocities when the separation between them is r ($r < r_0$)

When the masses are at distance r_0 from each other at rest their gravitational potential energy is given by

$$U_1 = -\frac{Gm_1m_2}{r_0}$$

As no external force acting on the system (gravitational attraction is the internal), the linear momentum of the system will remain conserved hence according to law of conservation of linear momentum we have

Final momentum = Initial momentum

Or $m_1v_1 + m_2v_2 = 0$

Or $$v_2 = -\frac{m_1v_1}{m_2} \quad \text{-------------------- (1)}$$

Now as no non-conservative forces are acting on the system the total mechanical energy will remains conserved hence

Final energy = initial energy

or Final kinetic energy + final potential energy = initial Kinetic energy + initial PE

or $$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{r} = 0 + 0 - \frac{Gm_1m_2}{r_0}$$

or $$m_1v_1^2 + m_2v_2^2 = 2Gm_1m_2\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

Substituting the value of v2 from equation (1) we have

$$m_1v_1^2 + m_2\left(\frac{m_1v_1}{m_2}\right)^2 = 2Gm_1m_2\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

Or $$v_1^2\left(1 + \frac{m_1}{m_2}\right) = 2Gm_2\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

Gives $$v_1 = m_2\sqrt{\frac{2G}{m}\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$ [$m_1 + m_2 = m$ given]

Substituting this in equation 1 we have

$$v_2 = m_1\sqrt{\frac{2G}{m}\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$