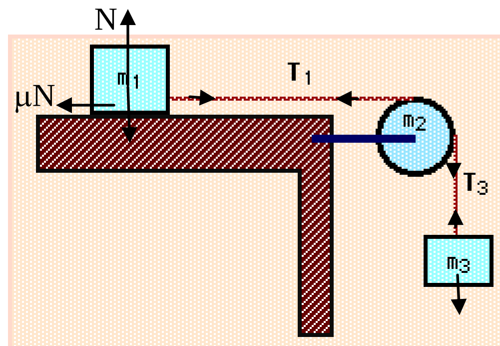
Q- A block of mass m_1 = 1 kg rests on a table with which it has a coefficient of friction μ = 0.54. A string attached to the block passes over a pulley to a block of mass m_3 = 3 kg. The pulley is a uniform disk of mass m_2 = 0.4 kg and radius 15 cm. As the mass m_3 falls, the string does not slip on the pulley.

a) With what acceleration does the mass m_3 fall?

The system will move because of the weight of m_3, the friction between the string and the pulley makes the pulley rotate and the tension in the string on either side on the pulley will be different.
Let the acceleration of the blocks be a and thus the angular acceleration α of the pulley will be a/R, where R is the radius of the pulley. The moment of inertia of the pulley about its own axis is $I = \frac{1}{2} m_2R^2$. The normal reaction N of the table is equal to the weight of m_1 and the friction force on the block will be $\mu m_1 g$.



Now writing the equations of motion for the three parts of the system we get

$T_1 - \mu m_1 g = m_1 * a$ ---- (1)

$(T_3 - T_1)*R = I*\alpha = \frac{1}{2} m_2R^2*(a/R)$

Or $(T_3 - T_1) = 0.5\ m_2*a$ ---- (2)

And $m_3g - T_3 = m_3*a$ ---- (3)

Adding the three equations we have

$m_3g - \mu m_1 g = [m_1 + 0.5\ m_2 + m_3]*a$

or

$$a = \left(\frac{m_3 - \mu m_1}{m_1 + 0.5 m_2 + m_3}\right) g = \left(\frac{3 - 0.54*1}{1 + 0.5*0.4 + 3}\right)*9.8 = 5.74 \text{ m/s/s}$$

Answer: a = 5.74 m/s^2

b) What is the tension in the horizontal string, T_1?

$T_1 - \mu m_1 g = m_1*a$ ---- (1)

Or $T_1 = \mu m_1 g + m_1*a$ =0.54*1*9.8 + 1*5.74 = 11.032 N

Answer: T_1 = 11.032 N

c) What is the tension in the vertical string, T_3?

$m_3g - T_3 = m_3*a$ ----- (3)

or $T_3 = m_3g - m_3*a$ = 3(9.8 – 5.74) = 12.18 N

Answer: T_3 = 12.18 N