Q- A disk of mass M_1 = 350 g and radius R_1 = 10 cm rotates about its symmetry axis at $f_{initial}$ = 157 rpm. A second disk of mass M_2 = 261 g and radius R_2 = 6 cm, initially not rotating, is dropped on top of the first. Frictional forces act to bring the two disks to a common rotational speed f_{final}. (a) What is f_{final}?

As there is no other force mentioned, we will consider the two disks as a system and the torque due to the frictional forces between them can be considered as internal torques. As no external torque is acting on the system, its angular momentum will remain conserved. Hence according to this law of conservation of angular momentum we have (ω for angular velocity and I for moment of inertia)

$I_1 * \omega_{initial} = (I_1 + I_2) * \omega_{final}$

Or $\omega_{final} = \left(\frac{I_1}{I_1 + I_2}\right)\omega_{initial}$

Now the moment of inertia of a circular disk about its axis is I = ½ mR^2 we have
(As there are ratios of the similar quantities, we need not the change the units.)

$$\omega_{final} = \left(\frac{0.5M_1R_1^2}{0.5M_1R_1^2 + 0.5M_2R_2^2}\right)\omega_{initial} = \left(\frac{350*100}{350*100 + 261*36}\right)157 = 123.77 \text{ rpm}$$

Answer: f_{final} = 123.77 rpm

(b) In the process, how much kinetic energy is lost due to friction?

The loss in kinetic energy of rotation is the difference in initial and final kinetic energy, hence

KE_{lost} = ½ $I_1*(\omega_{initial})^2$ - ½ $(I_1 + I_2) *(\omega_{final})^2$

Or $KE_{lost} = \frac{1}{2}*\frac{1}{2}M_1R_1^2\omega_{initial}^2 - \frac{1}{2}\left(\frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2\right)\omega_{finsl}^2$

Or

$KE_{lost} = \frac{1}{4}\left(0.350*0.10^2 + 0.261*0.06^2\right)\left(157*2\pi*60\right)^2 - \frac{1}{4}\left(0.350*0.10^2\right)\left(123.77*2\pi*60\right)^2$

Or $KE_{lost} = \frac{1}{4}\left(4.44*10^{-3}*3.5*10^9\right) - \frac{1}{4}\left(3.5*10^{-3}*2.175*10^9\right)$

$= 3.89*10^6 - 1.90*10^6 = 1.99*10^6$ J

Answer: $|KE_{lost}| = 1.99*10^6$ J