1. A hydraulic cylinder needs to push 5.5 gallons of fluid (water) per minute at 2000 psi (like pushing the fluid out of a syringe). How big is the outlet port where the fluid leaves the cylinder?

Answer:

Let the cross-section area of the rod is A_1 and that of the out let port is A_2. Let the velocity of the fluid in the two portions be v_1 and v_2 respectively.

Equation of continuity;
As the fluid in incompressible the volume of the fluid entering in a particular volume of the tube, in a given time interval must be equal and hence we have the volume of the fluid flown in time Δt

$$\Delta Q = A_1V_1\Delta t = A_2V_2\Delta t$$

Or the rate of the volume of the fluid flowing is given by

$$\Delta Q/\Delta t = A_1V_1 = A_2V_2 \quad \text{.....................} \ (1)$$

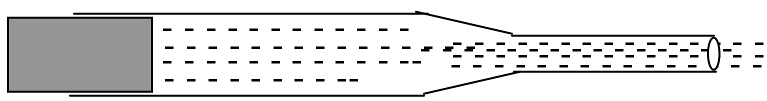
The equation $A_1V_1 = A_2V_2$ is called equation of continuity.

Considering the non-viscous incompressible fluid the pressure and the velocity of the fluid at different points can be given by Bernoulli's equation.

Bernoulli's equation for flow of an ideal liquid (non-viscous, incompressible) can be stated in different ways

1. if we consider energy of a mass m of the fluid, having volume V, v is the flow velocity at height h, then we can write
   $PV + m\,g\,h + (1/2)\,mv^2 = \text{constant}$ (for all points).
2. If the energy per unit volume is considered then the same equation is written as $P + \rho\,g\,h + (1/2)\,\rho v^2 = \text{constant}$; (dividing equation 1. by V) where ρ is the density of the fluid, and
3. If the pressure and velocity is measured in terms of height (heads) then
   $P/(\rho g) + h + v^2/2gh = \text{constant}$ (dividing equation 2. by ρg)

According to the given data we have to choose the equation.



Using the second equation for the inlet and the outlet ports

$$P_1 + \rho\,g\,h_1 + (1/2)\,\rho v_1^2 = P_2 + \rho\,g\,h_2 + (1/2)\,\rho v_2^2$$

If the average height of the two ports (horizontal position) are same $h_1 = h_2$ and the equation becomes

$$P_1 + (1/2)\,\rho v_1^2 = P_2 + (1/2)\,\rho v_2^2$$

Or $\quad (1/2)\,\rho(v_2^2 - v_1^2) = (P_1 - P_2) = \Delta P$ (say)

Substituting the value of v_2 from equation 1 we have

$$(1/2)\ \rho\{(A_1 v_1/A_2)^2 - v_1^2\} = \Delta P$$

Or $$(1/2)\ \rho v_1^2\ \{(A_1/A_2)^2 - 1\} = \Delta P$$

Or $$v_1 = \sqrt{\frac{2\Delta P A_2^2}{\rho(A_1^2 - A_2^2)}}$$

This is the standard form (may see Venturi Meter)

Now come to our problem, if the area of outlet port A_2 is much smaller then A_1, A_2^2 can be neglected as compared to A_1^2 and the equation becomes

$$v_1 = \sqrt{\frac{2\Delta P A_2^2}{\rho A_1^2}}$$

Gives $$A_2^2 = \frac{\rho A_1^2 v_1^2}{2\Delta P} = \frac{\rho(\Delta Q/\Delta t)^2}{2\Delta P}$$

or $$A_2 = \sqrt{\frac{\rho}{2\Delta P}} * (\Delta Q/\Delta t)$$

Now

1 psi = 1 lbf/in² = 6,894.75729 Pa
1 gallon =231 in^3 = 3.785 L = $3.785*10^{-3}$ m^3.

The pressure at the outlet is equal to the atmospheric pressure = 14.223 psi hence

$$\Delta P = 2000 - 14.223 = 1985.777 \text{ psi} = 13.69*10^6 \text{ Pa}$$

$$\Delta Q/\Delta t = 5.5*3.785*10^{-3}/60 = 3.47*10^{-4}\ m^3/s.$$

Consider water as a fluid then

$$\rho = 1000 \text{ kg/m}^3$$

Gives

$$A_2 = \sqrt{\frac{\rho}{2\Delta P}} * (\Delta Q/\Delta t) = \sqrt{\frac{1000}{2*13.69*10^6}} * (3.47*10^{-4}) = 2.097*10^{-6} m^2 = 2.097 mm^2$$
