Q- Four long straight wires each carrying current I are placed at the corners of a square of side a. The wires are parallel to each other and normal to the plane of the square. The direction of the current in two adjacent wires is out of the page and that in the other two is into the page. Find magnetic field at the center of the square.

The diagonal of the square will have a length $\sqrt{a^2+a^2}=\sqrt{2}*a$ and hence the distance of the center of the square from each wire is $\frac{a}{\sqrt{2}}$.

The magnitude of magnetic field due to a long straight wire carrying current I at a distance r from it is given by

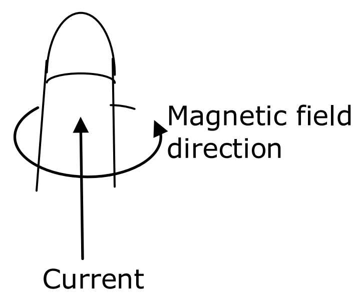
$$B=\frac{\mu_0 I}{2\pi r}$$

The direction of magnetic field due to a long straight wire carrying current I is given by the right hand thumb rule. According to this rule close the fist of right hand with the thumb stretched away. If the thumb shows the direction of the current in the wire the curled fingers will show the direction of the magnetic field lines.

Now as the current in all four wires is same in magnitude as well as the distance of the center equal the magnitude of field due to all wires will be equal and is given by

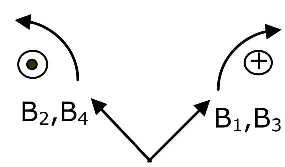
$$B=\frac{\mu_0 I}{2\pi r}=\frac{\sqrt{2}*\mu_0 I}{2\pi a} \qquad \text{----- (1)}$$

{substituting the value of r.}



For the first wire the current is out of the page and hence keeping the thumb of right hand outward the fingers are curled anticlockwise from the top and hence field at the center will be in the direction from center to wire 2. Resolving B along x and y direction we have field due to first wire

$$\vec{B}_1=B\cos 45^0\,\hat{i}+B\sin 45^0\,\hat{j}=\frac{B}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)=\frac{\mu_0 I}{2\pi a}\left(\hat{i}+\hat{j}\right)$$



For the second wire the current is in to the page and hence keeping the thumb of right hand inward, the fingers are curled clockwise from the top and hence field at the center will be in the direction from center to wire 1. Resolving B along x and y direction we have field due to second wire

$$\vec{B}_2=B\cos 135^0\,\hat{i}+B\sin 135^0\,\hat{j}=\frac{B}{\sqrt{2}}\left(-\hat{i}+\hat{j}\right)=\frac{\mu_0 I}{2\pi a}\left(-\hat{i}+\hat{j}\right)$$

For the third wire the current is in to the page and hence keeping the thumb of right hand inward the fingers are curled clockwise from the top and hence field at the center will be in the direction from center to wire 2. Resolving B along x and y direction we have field due to third wire

$$\vec{B}_3=B\cos 45^0\,\hat{i}+B\sin 45^0\,\hat{j}=\frac{B}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)=\frac{\mu_0 I}{2\pi a}\left(\hat{i}+\hat{j}\right)$$

And lastly for the fourth wire the current is out of the page and hence keeping the thumb of right hand outward the fingers are curled anticlockwise from the top and hence field at the center will be in the direction from center to wire 1. Resolving B along x and y direction we have field due to fourth wire

$$\vec{B}_4 = B\cos 135^0\,\hat{i} + B\sin 135^0\,\hat{j} = \frac{B}{\sqrt{2}}\left(-\hat{i} + \hat{j}\right) = \frac{\mu_0 I}{2\pi a}\left(-\hat{i} + \hat{j}\right)$$

Hence the resultant magnetic field due to all wires at the center of the square is given by adding all the four vectors. This results in cancellation of the fields in x direction and adding up the fields in y direction and hence the net field will be

$$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 I}{2\pi a}\left(0\hat{i} + 4\hat{j}\right) = \frac{2\mu_0 I}{\pi a}\left(\hat{j}\right)$$

Substituting the numerical values we have

$$\vec{B}_{net} = \frac{2\mu_0 I}{\pi a}\left(\hat{j}\right) = \frac{2*4\pi*10^{-7}*20}{\pi*0.2}\left(\hat{j}\right) = 8.0*10^{-5}\left(\hat{j}\right)\text{ T}$$

Hence the field at the center is $8.0*10^{-5}$ Tesla parallel to y direction.