Q- Using kinematics, you will be asked to calculate the initial velocity of the projectile by measuring the vertical drop, h, and the horizontal distance from the launcher, d, assuming the initial direction of the displacement is horizontal. Using the equations: $x_f = x_0 + v_{x0}t$ and $y_f = y_0 + v_{y0}t - 1/2gt^2$, derive an expression for this initial velocity in terms of h and d only (no other variable in the expression).

Answer:

Let the initial velocity of the projectile be v_0, in horizontal direction.

The initial vertical velocity is zero and hence the relation gives the vertical displacement in time t is

$y_f = y_0 + v_{y0}t - 1/2gt^2$

or $y_f - y_0 = 0 - 1/2gt^2$

But the vertical drop $y_f - y_{0\,=}$ - h (negative because downwards) hence we have

$h = ½\ g*t^2$ ---------- (1)

Now in the same time the horizontal distance traveled (constant velocity in horizontal direction $=v_0$)

$d = v_0*t$ ---------- (2)

Substituting for t from equation 2 in equation 1 we have

$$h = \frac{1}{2} * g * \left(\frac{d}{v_0} \right)^2 = \frac{g * d^2}{2 * v_0^2}$$

or $$v_0^2 = \frac{g * d^2}{2 * h}$$

gives $$v_0 = \sqrt{\frac{g * d^2}{2 * h}} = d\sqrt{\frac{g}{2 * h}}$$

This is the required relation