Q- Isotope 137 Cs of Cesium has a half life of 30 years. Suppose a distributor sells 5 grams of it in January 1, 2000. a) Write an equation with giving the number of grams of the isotope left "t" years after January 1, 2000. b) How many grams are present after 6 years?

Reading

According to the law of radioactive disintegration the rate of disintegration dN/dt (known as activity as well) is directly proportional to the number of atoms present in the sample hence

$$\frac{dN}{dt} = -\lambda N$$

Here λ is the constant of proportionality, called disintegration constant and having different values for different nuclei. The negative sign is because the number N is decreases with time and hence the rate of change of N is negative.

Above equation can be written as

$$\frac{dN}{N} = -\lambda * dt$$

If initially (t=0) number of the radioactive nuclei present in the sample are N_0, then integrating this equation with proper limits we get the number nuclei present N as a function of t as

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{0}^{t} dt \quad \text{Gives} \qquad N = N_0 * e^{-\lambda t} \qquad \text{---------- (1)}$$

The half life is the time in which the number of nuclei in sample becomes half of the initial value i.e. $N_0/2$ and substituting it in equation (1) we have

$$\frac{N_0}{2} = N_0 * e^{-\lambda t}$$

Or ½ = $e^{-\lambda t}$ Or $\ln 2 = \lambda t$ Or $t = t_{1/2} = \ln 2/\lambda$ ——-------- (2)

Now our problem

(a) Using equation 2 as half-life of our sample is 30 years we have

$\lambda = \ln 2/t_{1/2} = 0.6931/30$ = per year

If the mass of the sample in m (in grams) then the number of atoms present is

$N = (m/M)*N_A$

Where M is the molar mass (gram/ mole) and N_A is the Avogadro number hence equation (1) can be written as (M and N_A get cancelled from both sides)

$$m = m_0 * e^{-\lambda t}$$

(b) The mass of the nuclei remaining after time t = 6 years is given by above equation as

or $$m = 5.0 * e^{-\frac{0.6931*6}{30}} = 5 * e^{-0.1386} = 5 * 0.8706 = 4.3529 \text{ gm}$$

Hence the mass of the Cesium 137 in the sample after 6 years is 4.3529 gm.