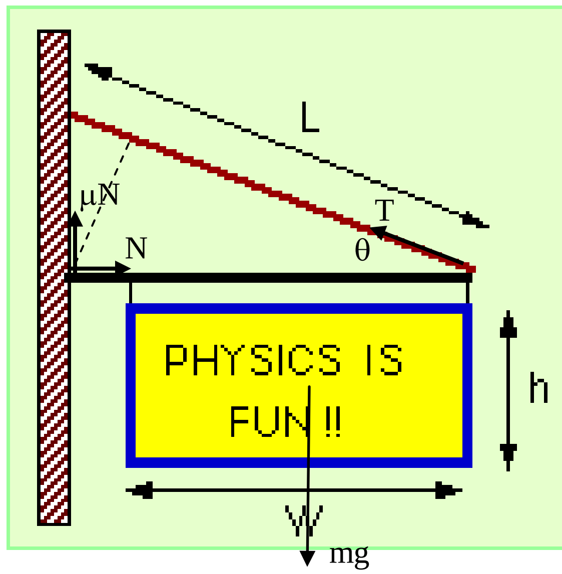
Q- A rectangular signboard has a mass of 1050 kg, a height h = 1 m, and a width W = 4 m. It is held by a light rod of length 5 m that is perpendicular to a rough wall. A guy wire at $30^0$ to the horizontal holds the sign to the wall. Note that the distance from the left edge of the sign to the wall is 1 m. Suppose we rely upon friction between the wall and the rod to hold up the sign (there is no hinge attaching the rod to the wall). What is the smallest value of the coefficient of friction such that the sign will remain in place?

Let the end of the rod is just at the verge of slipping. The tension in the guy wire is T. The horizontal and vertical components of the tension are T*cosθ and T*sinθ. Let the normal reaction of the wall is N, then the limiting friction force on the rod will be μN in upward direction as shown in the figure.




Now as the sign is in equilibrium, balancing the horizontal and vertical forces on the rod gives

$$N - T^*\cos\theta = 0 \quad \text{------ (1)}$$

And $$\mu N + T^*\sin\theta - mg = 0 \quad \text{------- (2)}$$

Now for equilibrium the net torque on the system about any axis must be zero hence taking the torques (product of force and perpendicular distance from the axis of rotation) about the point of contact we get

$$T^*5^*\sin\theta - mg(1+2) = 0$$

Or $$T = 3mg/(5\sin\theta) \text{----- (3)}$$

[1. We have chosen the point of contact as the axis of rotation because the torque due to N and μ N about this axis is zero.
2. The tension T along the guy wire is along the wire and hence perpendicular distance of the line of action of T is 5*sinθ shown by the dotted line.
3. The weight of the sign is acting at its center of mass which is mid way and hence its distance from the point of contact is half of the width plus the one meter on side hence total 3 m.]
Solving equation (1) and (2)

$$\mu T\cos\theta + T\sin\theta = mg$$

or $$T(\mu\cos\theta + \sin\theta) = mg$$

Substituting value of T from equation (3) this will give

$$\frac{3mg}{5^*\sin\theta}\left(\mu\cos\theta + \sin\theta\right) = mg$$

Or $$3\mu\cos\theta = 2\sin\theta$$

Or $$\mu = (2/3)\tan\theta = (2/3)\tan 30^0 = 2/(3\sqrt{3}) = 0.38$$

Thus the coefficient should be greater than 0.38.