Q- Find the required mass of block A so that when it is released from rest it moves block B by 1 m in 2 sec. Block B weights 60 N. Pulleys are massless and no friction. The angle θ is 30^0.

Answer:

Upper two pulleys are fixed and the lowest is movable.

The same string is passing through the pulleys and hence the tension T in it is same everywhere (considering the pulleys are mass less and frictionless).

Three parts of the string are supporting the lower pulley and the block A and hence the total upward force on the lower pulley and block A is 3T. Let the mass of A is m_A and its acceleration is a, then equation of motion for block A can be written as

$$m_A g - 3*T = m_A a \qquad \text{---------- (1)}$$

Now if the block A is displaced down by x then the additional string required between the pulleys is 3x (three strings connecting), and hence the length of the string between upper pulley and B will reduced by 3x. As the ratio of acceleration is same as ratio of displacement, acceleration of block B will be 3a.

May be done in this way

If x is the distance of A from the center of pulley E to D then

Total free length of the string is

$$l = 3x - CE + y_B$$

Differentiating the equation twice wrt t and as l is constant we have

$$0 = 3(d^2x/dt^2) - 0 + (d^2y_B/dt^2)$$

Or acceleration of B is three times of A. negative sign is because of the direction.

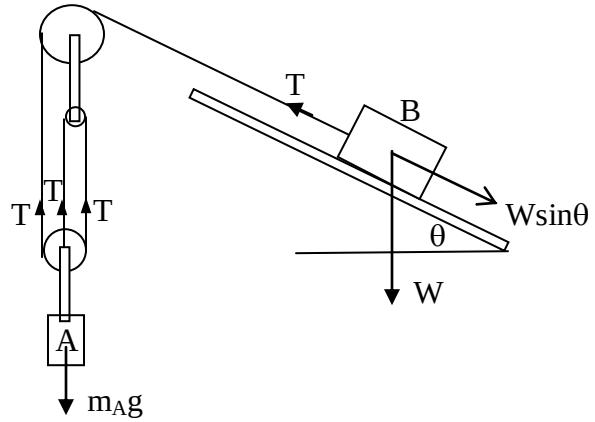
Mass of B $m_B = W/g = 58.86/9.81 = 6.00$ kg

Hence equation of motion of B up the incline can be written as

$$T - W*\sin\theta = m_B*3a \qquad \text{---------- (2)}$$

Hence solving the two equations we have

$m_A g - 3\ W*\sin\theta = (m_A + 9m_B)*a$ ---------- (3)



or $$m_A = \frac{9m_B a + 3W\sin\theta}{g - a}$$

Now the distance moved by B up the incline is 1 m in 2 s from rest we have

[$s = ut + \frac{1}{2} at^2$]

$1 = 0 + 0.5*3a*2^2$

Gives a = (1/6) m/s^2

Substituting values in equation 3 we get

$$m_A = \frac{9m_B a + 3W\sin\theta}{g - a} = \frac{9*6.0*(1/6) + 3*58.86*0.5}{9.81 - (1/6)} = 10.1 \text{ kg.}$$

Hence mass of block A must be 10.1 kg.