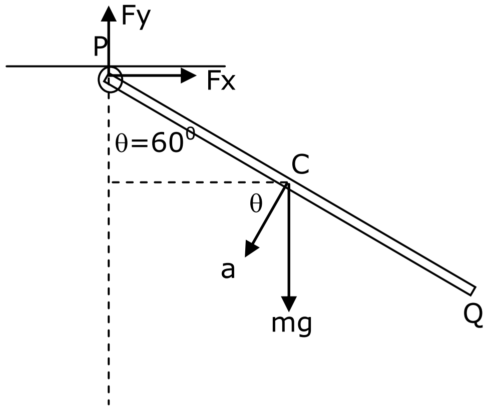
Q- A uniform rod PQ, mass m and length *l* is free to rotate in a vertical plane about a fixed horizontal axis passing through P. The rod is released from rest when PQ makes an angle 60^0 with the downward vertical. Find the magnitude of the force exerted by the rod on the pivot just after it is released.

The force on the pivot at axis of rotation will be equal and opposite of the force on the rod by the pivot.



The force on the rod due to the pivot is neither in horizontal not in vertical direction but we can resolved it along these directions.

Let the components are F_x and F_y respectively as in figure. These forces and the weight of the rod mg are the external forces on the rod and accelerating it in such a way that the rod starts rotating about the axis.

To find the acceleration of center of mass of the rod C, the easy way is to find the angular acceleration of the rod.

The torque on the rod about the axis of rotation (P) is only due to mg as the other forces are passing through the axis and their moment is zero. Hence

$$\tau = \left|\vec{r} \times \vec{F}\right| = (PC) * mg * \sin\theta = mg(l/2)\sin\theta$$

Moment of inertia of the rod about the axis of rotation is (using parallel axis theorem)

$$I = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

Hence the angular acceleration of the rod at this moment will be

$$\alpha = \frac{\tau}{I} = \frac{mgl\sin\theta}{2} * \frac{3}{ml^2} = \frac{3g\sin\theta}{2l}$$

And hence the acceleration of center of mass C will be

$$a = \alpha * \frac{l}{2} = \frac{3g\sin\theta}{4}$$

The direction of this acceleration is perpendicular to the rod and hence resolving it we get the acceleration of point C in horizontal and vertical directions as – a*cosθ and – a*sinθ respectively (negative sign according to the directions).

Now applying Newton's law of motion (F = ma) we can write equations of motion for center of mass C in horizontal and vertical direction as

$$F_x = m * (-a\cos\theta) = -\frac{3mg\sin\theta\cos\theta}{4} = -\frac{3\sqrt{3} * mg}{16}$$

And $$F_y - mg = m * (-a\sin\theta) = -\frac{3mg\sin^2\theta}{4} = -\frac{9 * mg}{16}$$

Or $\qquad F_y = mg - \dfrac{9 * mg}{16} = \dfrac{7mg}{16}$

Hence the net force on the rod due to the pivot will be

$$\vec{F} = -\frac{3\sqrt{3}mg}{16}\hat{i} + \frac{7mg}{16}\hat{j} \qquad \text{[i and j are unit vectors]}$$

And its magnitude will be

$$F = \sqrt{F_x^2 + F_y^2} = \frac{mg}{16}\sqrt{27+49} = \frac{mg}{8}\sqrt{19} = 0.545 * mg$$

And hence the force by the rod on the pivot will have same magnitude of 0.545 mg.