Q- In a long wire of radius a = 3.1 mm the current density varies as distance r as j (r) = j_0 (r/a). Here r is the distance from axis and j_0 = 310 A/m^2 is a constant. Find magnetic field at (a) on the axis of wire (b) at distance a/2 from the axis and (c) at the surface of the wire.

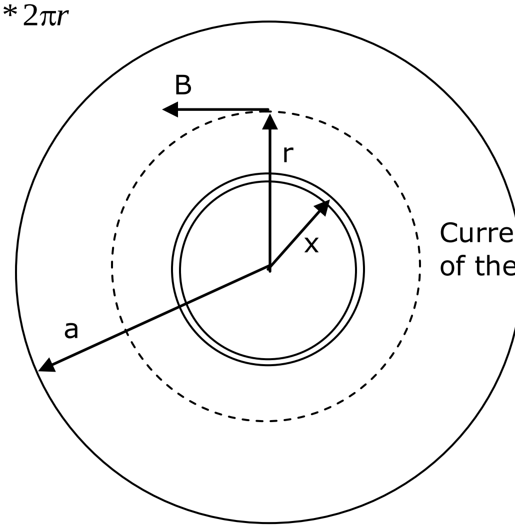
The ampere's circuital law gives the magnitude of the magnetic field B for a symmetric current distribution and is given as

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$

Here the first term gives the line integration of the magnetic field for a closed loop and the I is the current within the loop.

Inside the wire as the current is uniformly distributed about the axis, the field will be radially symmetric in magnitude and the direction will be tangential (θ = 0). The magnetic lines of force will be circular with the center at the axis of the wire. Hence considering a circular loop, with center at the axis of the wire the magnitude B of the magnetic field will be constant and can be taken out of the line integral. Then the rest of the integral will be the summation of the length of the closed loop and will be equal to 2πr. Thus for a circular loop of radius r the line integral will be given by

$$\oint \vec{B} \bullet d\vec{l} = \oint B * dl * \cos\theta = B\oint dl = B * 2\pi r$$



Now to calculate the current within the loop consider a thin ring between the radii x and x + dx with the center at the axis.
Area of this ring will be dA = 2πx*dx and hence the current in this infinitesimally thin ring will be

dI = J*dA = $J_0 * \frac{x}{a} * 2\pi x * dx$

Hence total current within the loop of radius r will be

I = $\int dI = \frac{2\pi J_0}{a}\int_0^r x^2 * dx = \frac{2\pi J_0 r^3}{3a}$

Substituting the values in equation of ampere's law we have

$$B * 2\pi r = \frac{\mu_0 2\pi J_0 r^3}{3a}$$

Gives B as a function of r

$$B = \frac{\mu_0 J_0 r^2}{3a}$$

Now consider the different cases.

(a) For r = 0, B = 0. We have to consider the axis of the wire and hence the area for the loop is zeroing the current in the loop is zero and hence the magnetic field will be zero.

(b) Substituting r = a/2 and the other quantities we have

$$B = \frac{\mu_0 J_0 r^2}{3a} = \frac{\mu_0 J_0 (a/2)^2}{3a} = \frac{\mu_0 J_0 a}{12} = \frac{4\pi * 10^{-7} * 310 * 3.1 * 10^{-3}}{12}$$

$= 1.01*10^{-7}$ T

(c) Substituting r =a we have

$$B = \frac{\mu_0 J_0 r^2}{3a} = \frac{\mu_0 J_0 a^2}{3a} = \frac{\mu_0 J_0 a}{3} = \frac{4\pi * 10^{-7} * 310 * 3.1 * 10^{-3}}{3}$$

$= 4.025*10^{-7}$ T