

Q- A uniform solid disk rolls down an inclined plane. Find the minimum coefficient of static friction that will permit the disk to roll without slipping, if the angle of the incline is 30° with the horizontal.

Let the mass of the disk is m and the radius is R .

The forces acting on the disk are its weight mg , the normal reaction of the surface N and the friction force F .

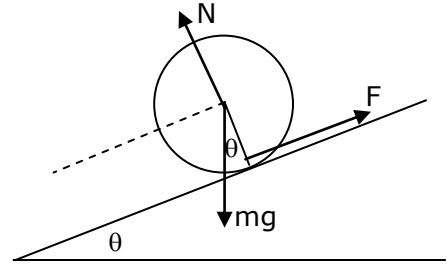
Let the coefficient of friction μ is such that the friction is just sufficient to make it roll and the disk is at the verge of sliding. Resolving the weight mg , along tangential and normal to the incline directions, as there is no motion (of the center of mass) in the direction normal to the incline the net forces in normal direction must be zero hence

$$N - mg \cos \theta = 0$$

Gives $N = mg \cos \theta$

And if the tangential acceleration is a then writing equation of motion

$$mg \sin \theta - F = ma$$



but as the disk is at the verge of sliding the friction will be limiting and will be μN hence

$$mg \sin \theta - \mu N = ma$$

or $mg \sin \theta - \mu mg \cos \theta = ma$ {substituting N }

Gives $a = g \sin \theta - \mu g \cos \theta$ ----- (1)

Now if the disk is having rolling then its angular acceleration is given by

$$\alpha = a / R$$

Hence writing the equation of rotational motion about the horizontal axis of disk we have

$$\tau = I * \alpha$$

Where τ is the torque which is $F * R$ and I is the moment of inertia of the disk about its axis which is $\frac{1}{2} mR^2$. Hence

$$F * R = \frac{1}{2} mR^2 * \alpha$$

Or $\mu mg \cos \theta * R = \frac{1}{2} mR^2 * \frac{a}{R}$

Gives $a = 2\mu g \cos \theta$ ----- (2)

From equations 1 and 2 we have

$$g \sin \theta - \mu g \cos \theta = 2\mu g \cos \theta$$

Gives $\mu = \frac{\tan \theta}{3} = \frac{\tan 30^\circ}{3} = \frac{1}{3\sqrt{3}} = 0.192$