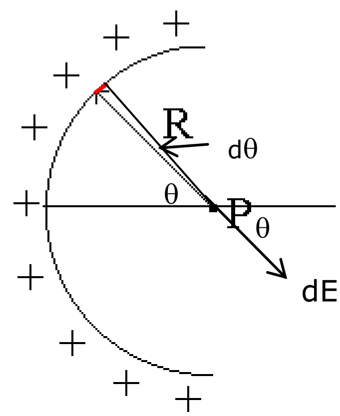
A half-ring with a radius 9 cm has a total charge 10 μC uniformly distributed along its length.
(a) What is the linear charge density on the ring?
(b) What is the magnitude of the electric field at the center of the half-ring?



(a) The length of the half ring will be given by

L = πR = 3.14*0.09 m = 0.28 m

Hence the charge per unit length λ = Q/L = $10*10^{-6}/0.28$ = **$35.7*10^{-6}$ C/m**

(b) As the charge is symmetrically distributed about x axis the field in y and z direction has a resultant component zero and hence only the components in x directions are to be added.

Consider an infinitesimally small element subtending angle dθ at the center of the ring P as in diagram. The length of the ring will be R*dθ and hence charge on the element will be

dQ = λ*Rdθ

Hence the field due to the element charge will be

$$dE = \frac{\lambda R * d\theta}{4\pi \in_0 R^2} = \frac{\lambda * d\theta}{4\pi \in_0 R}$$

Component of this field in x direction will be

$$dE_x = dE\cos\theta = \frac{\lambda * \cos\theta * d\theta}{4\pi \in_0 R}$$

Hence field in x direction will be

$$E_x = \int dE_x = \frac{\lambda}{4\pi \in_0 R}\int_{-\pi/2}^{\pi/2}\cos\theta * d\theta = \frac{\lambda}{4\pi \in_0 R}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} = \frac{\lambda}{4\pi \in_0 R} * 2$$

Or $$E_x = \frac{\lambda}{4\pi\epsilon_0 R} * 2 = \frac{9*10^9*35.7*10^{-6}*2}{0.09} = 7.14 * 10^6 \text{ N/C}$$

Due to symmetry of the charge distribution Ey and Ez both are zero hence the total field at P will be that in x direction and is

E = $7.14*10^6$ N/C