

Q- The tenth brightest member of the cluster of galaxies known as Abell1185 has a brightness of $2.70 \times 10^{10} \text{ Wm}^{-2}$. A line in its spectrum at a wavelength of 503.1 nm is identified as being a member of the Balmer series that has a rest wavelength of 486.1 nm. Let the luminosity of the tenth brightest galaxy in any cluster is $1.00 \times 10^{41} \text{ W}$, the speed of light c is $3.00 \times 10^5 \text{ km/s}$ and $1 \text{ Mpc} = 3.09 \times 10^{22} \text{ m}$.

a) Calculate the distance of the Abell1185 cluster of galaxies in metres and in mega-parsecs.

The radiant power P from a star (or galaxy) is the energy emitted by the star (or galaxy) per unit time (W). This energy is distributed in all direction and hence the intensity I (The amount of energy incident per unit area per unit time Wm^{-2}) at a distance r is given by

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

This gives the distance of the galaxy from the earth as

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{2.70 \times 10^{10}}{4 \times 3.14 \times 1.0 \times 10^{-41}}} = 1.47 \times 10^{25} \text{ m}$$

The distance will be given in mega parsecs as

$$r = \frac{1.47 \times 10^{25}}{3.09 \times 10^{22}} = 475.7 \text{ Mega parsecs.}$$

b) Calculate the redshift of the tenth brightest galaxy in the Abell1185 cluster.

The increase in the wavelength (or decrease in frequency) of light emitted by a receding source as compared to a stationary source is called red shift. Thus the red shift is given by

$$\Delta\lambda = \lambda' - \lambda = 503.1 - 486.1 = 17.0 \text{ nm.}$$

c) Calculate the speed of recession in m/s

When the source is receding away, the red shift is given by the formula

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Where v is the velocity of the source and c is the speed of light. λ is the rest wavelength of the light.

Hence the speed of the galaxy is given by

$$\frac{17}{486.1} = \frac{v}{3.0 \times 10^5}$$

$$\text{Or } v = \frac{17}{486.1} \times 3.0 \times 10^5 = 1.05 \times 10^4 \text{ km/s} = 1.05 \times 10^7 \text{ m/s}$$

d) Calculate the value of Hubble constant.

The mathematical expression for Hubble's Law is as follows:

$$v = H \cdot D$$

Where v is the recessional velocity, typically expressed in km/s. H_0 is Hubble's constant.

Hence the Hubble constant is given by

$$H = \frac{v}{D} = \frac{1.05 \cdot 10^4 \text{ km/s}}{1.47 \cdot 10^{22} \text{ km}} = 7.14 \cdot 10^{-19} \text{ s}^{-1}.$$
