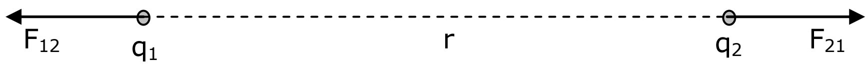
Q- Two point charges q_1 = (+) 1μC and q_2 = (+) 9μC are separated by a distance of 50cm. (a) Find the magnitude and direction of the Coulomb force on each charge due to the other. (b) At what point on the line joining the two charges should a charge q = (-) 1μC be placed so that the net force on q due to q_1 and q_2 is zero?
(c) Find the location of charge q for zero force if the charge q_2 is replaced by a charge of (-) 9 μC.

If two charges of magnitude q_1 and q_2 are separated by distance r then the force between them is given by Coulomb's law and according to this law, the force of interaction between two point charges is given by

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi \in_0 \in_r r^2} \hat{r}$$

Where F_{21} is the force on charge q_2 due to charge q_1 and $\hat{r}$ is the unit vector in the direction of the line joining from charge q_1 to q_2. $\in_0$ is a constant called the permittivity of free space and $\in_r$ is the relative permittivity (dielectric constant) of the medium. For air or vacuum, value of $\in_r$ is 1.0, $\frac{1}{4\pi \in_0}$ = $9*10^9$ Nm^2/C^2

Naturally the force F_{12} on charge q_1 due to q_2 will be equal in magnitude and opposite in the direction.



Now the force acting on charge q_2 due to charge q_1 will be

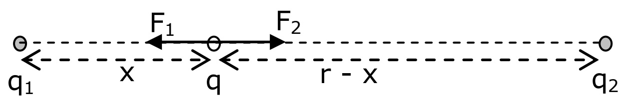
$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\,\hat{r} = 9 * 10^9 * \frac{1*10^{-6}*9*10^{-6}}{(50*10^{-2})^2} * \hat{r} = 0.324\,\hat{r}\,N$$

Hence the force on charge q_2 is 0.324 N and is towards right. Similarly

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}(-\hat{r}) = -\,9 * 10^9 * \frac{9*10^{-6}*1*10^{-6}}{(50*10^{-2})^2} * \hat{r} = -\,0.324\,\hat{r}\,N$$

Hence the force on charge q_1 is 0.324 N and is towards left.

(b) The force acting on a point charge due to number of point charges is given by the law of superposition of forces and hence given by the resultant of the forces due to all other charges. Thus the force on q is given by the resultant of the forces due to q_1 and q_2.



Now as charge q is negative and both the other are positive, both charges attracts q and hence the possibility for the forces on q to balance each other is only when q is between q_1 and q_2. Let q is placed at distance x from q_1 as in figure then force on it due to q_1 will be

$$\vec{F}_1 = \frac{q_1 q}{4\pi \in_0 x^2}\hat{r} = 9*10^9 \frac{1*10^{-6}*(-1*10^{-6})}{x^2}\hat{r} = \frac{9*10^{-3}}{x^2}(-\hat{r})$$

And the force due to charge q_2 will be

$$\vec{F}_2 = \frac{q_2 q}{4\pi \in_0 (r-x)^2}(-\hat{r}) = 9*10^9 \frac{9*10^{-6}*(-1*10^{-6})}{(r-x)^2}(-\hat{r}) = \frac{81*10^{-3}}{(r-x)^2}(\hat{r})$$

The negative sign of unit vector r is because q is on the left of q_2.

The distance x is such that the net force is zero, hence we have

$$\vec{F}_1 + \vec{F}_2 = 0$$

Or $$\frac{9*10^{-3}}{x^2}(-\hat{r}) + \frac{81*10^{-3}}{(r-x)^2}(\hat{r}) = 0$$

Gives $$\frac{(r-x)^2}{x^2} = 9$$

Or $$\frac{(r-x)}{x} = 3$$ [taking appropriate positive sign only]

Gives x = r/4 = 50/4 = 12.5 cm.

Hence q will experience zero force at 12.5 cm from q_1, towards q_2.

(c) If the charge q_2 is -9 μC, it will repel q and hence for zero force q should be on the same side of q_1 and q_2. If it is at right of both, q_2 (larger in magnitude) is nearer and F_2 will be always greater than F_1. Hence q must be on the left of both q_1 and q_2 as in figure. Force on q due to charge q_1 will be

$$\vec{F}_1 = \frac{q_1 q}{4\pi \in_0 x^2}(-\hat{r}) = 9*10^9 \frac{1*10^{-6}*(-1*10^{-6})}{x^2}(-\hat{r}) = \frac{9*10^{-3}}{x^2}(\hat{r})$$

The negative sign of unit vector r is because q is on the left of q_1.

And the force due to charge q_2 will be (the distance or q from q_2 is r + x)

$$\vec{F}_2 = \frac{q_2 q}{4\pi \in_0 (r+x)^2}(-\hat{r}) = 9*10^9 \frac{(-9*10^{-6})*(-1*10^{-6})}{(r+x)^2}(-\hat{r}) = \frac{81*10^{-3}}{(r+x)^2}(-\hat{r})$$ The negative sign is because q is on the left of q_2.

The distance x is such that the net force is zero, hence we have

$$\vec{F}_1 + \vec{F}_2 = 0$$

Or $$\frac{9*10^{-3}}{x^2}(\hat{r}) + \frac{81*10^{-3}}{(r+x)^2}(-\hat{r}) = 0$$

Gives $$\frac{(r+x)^2}{x^2} = 9$$

Or $$\frac{(r+x)}{x} = 3$$ [taking appropriate positive sign only]

Gives x = r/2 = 50/2 = 25.0 cm.

Hence q will experience zero force at 25.0 cm from q_1, away from q_2.