

Q- A non uniform electric field is given by the expression $E = ayi + bzj + cxk$, where a , b , and c are constants. Determine the electric flux through a rectangular surface in the xy plane, extending from $x = 0$ to $x = w$ and from $y=0$ to $y=h$.

The flux through a surface is given by the scalar product of the field strength and the area of the surface.

As the rectangular surface is in xy plane the flux corresponding to x and y component of field will be zero (parallel to the surface) and the flux through the surface is only due to z component of the field. The z component of the field is varying with x only and directly proportional to x coordinate.

Consider an infinitely thin strip of the surface at a distance x from y axis of thickness dx as in figure. Field at this strip will have magnitude $c*x$ and will be normal to the surface hence the flux through this strip will be given by

$$d\phi = \text{field} * \text{area}$$

or $d\phi = (c*x)(h*dx) = c h x * dx$

Hence flux through the whole rectangle will be

$$\phi = \int d\phi = \int_0^w chx * dx = ch \int_0^w x * dx = \frac{c * h * w^2}{2}$$

