Q- A non uniform electric field is given by the expression \( \mathbf{E} = ay\mathbf{i} + bz\mathbf{j} + cx\mathbf{k} \), where \( a, b, \) and \( c \) are constants. Determine the electric flux through a rectangular surface in the \( xy \) plane, extending from \( x = 0 \) to \( x = w \) and from \( y = 0 \) to \( y = h \).

The flux through a surface is given by the scalar product of the field strength and the area of the surface.

As the rectangular surface is in \( xy \) plane the flux corresponding to \( x \) and \( y \) component of field will be zero (parallel to the surface) and the flux through the surface is only due to \( z \) component of the field. The \( z \) component of the field is varying with \( x \) only and directly proportional to \( x \) coordinate.

Consider an infinitely thin strip of the surface at a distance \( x \) from \( y \) axis of thickness \( dx \) as in figure. Field at this strip will have magnitude \( c*x \) and will be normal to the surface hence the flux through this strip will be given by

\[
d\phi = \text{field*area}
\]

or

\[
d\phi = (c\times x)(h\times dx) = c\times h\times x\times dx
\]

Hence flux through the whole rectangle will be

\[
\phi = \int d\phi = \int_0^w chx \times dx = \int_0^w x \times dx = \frac{c\times h\times w^2}{2}
\]