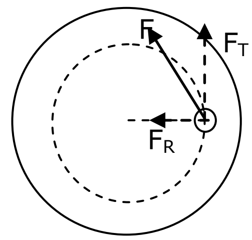
Q- A small coin is placed on the horizontal surface of a rotating disk at a distance r from the center. The disk starts from rest and is given a constant angular acceleration $\alpha$. The coefficient of static friction between the coin and the disk is $\mu_s$. Determine the number of revolution N through the disk turns before the coin slips.

As the disk rotates about its axis with it the coin will also rotates on a horizontal circular path of radius r. The friction is the only horizontal force acting on the coin which provides tangential acceleration to the coin and the centripetal acceleration to keep the coil moving on the circular path of radius r.

The disk starts rotating with constant angular acceleration $\alpha$ from rest and hence its angular velocity after time t will be given by

$$\omega = \alpha * t$$




It till then the coin is not sliding its angular velocity will also be the same $\omega$.

Tangential acceleration of the coin will be

$$a = \alpha * r$$

And hence magnitude of the tangential force $F_T$ on the coin will be constant and given by

$$F = m*a = m* \alpha *r$$

As the angular velocity of the coin increases with the disk the magnitude centripetal force (always towards the center of radial) increases with time and time t is given by

$$F_R = m*\omega^2*r$$

As both forces are perpendicular to each other magnitude of their resultant is given by

$$F = \sqrt{F_T^2 + F_R^2} = \sqrt{(m\alpha r)^2 + (m\omega^2 r)^2}$$

This force is produced by the friction and in limiting case equal to the limiting friction force equal to $\mu_s N = \mu_s mg$ because normal reaction is equal to weight mg.

Hence just before the coin slips, if the angular velocity of the disc and the coin is $\omega$ then

$$\mu_s mg = \sqrt{(m\alpha r)^2 + (m\omega^2 r)^2}$$

Or $\quad (\mu_s g)^2 = \alpha^2 r^2 + \omega^4 r^2$

Gives $\quad \omega^4 = \frac{\mu_s^2 g^2}{r^2} - \alpha^2$

Or $\quad \omega^2 = \sqrt{\frac{\mu_s^2 g^2}{r^2} - \alpha^2}$ ------------------- (1)

Now analogues to the second equation of motion [$v^2 = u^2 + 2as$] we may write the equation of motion for circular motion and the angle turned $\theta$ by the disk before the coin slips (angular velocity increases from 0 to $\omega$) is given by

$$\omega^2 = 0 + 2\alpha\,\theta$$

Using equation 1 we have

$$\theta = \frac{1}{2\alpha}\sqrt{\frac{\mu_s{}^2 g^2}{r^2} - \alpha^2}$$

Hence number of revolutions of the disk is given by

$$N = \theta/2\pi = \frac{1}{4\pi\alpha}\sqrt{\frac{\mu_s{}^2 g^2}{r^2} - \alpha^2}$$