

Q- A rocket rises vertically from rest, with an acceleration of  $3.2 \text{ m/s}^2$  until it runs out of fuel at an altitude of  $1200 \text{ m}$ . After this point, its acceleration is that of gravity, downward.

(a) What is the velocity of the rocket when it runs out of fuel?

Initial velocity of the rocket  $u = 0$

Acceleration of the rocket  $a = 3.2 \text{ m/s}^2$

[upward hence positive]

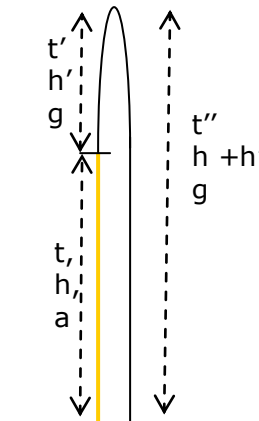
Displacement during the first lap  $h = 1200 \text{ m}$

Final velocity of the rocket (say)  $v = ?$

Using the third equation of motion we have

$$v^2 = 0 + 2*a*h$$

Or  $v = \sqrt{2ah} = \sqrt{2*3.2*1200} = 87.64 \text{ m/s}$



(b) How long does it take it to reach this point?

Time taken  $t$  to reach point is given by the second equation of motion as

$$s = ut + \frac{1}{2} at^2$$

or  $h = 0 + \frac{1}{2} a*t^2$

or  $t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2*1200}{3.2}} = 27.39 \text{ s}$

(c) What maximum altitude does the rocket reach?

After  $1200 \text{ m}$  of height it will rise up till its velocity becomes zero and hence the height reached after it runs out of the fuel  $h'$  will be given by the third equation as

$$[v^2 = u^2 + 2*a*s]$$

Or  $0 = (87.64)^2 + 2*(-9.8)*h'$

Gives  $h' = 7680/19.6 = 391.84 \text{ m}$ .

Hence maximum altitude reached by the rocket will be

$$h + h' = 1200 + 391.84 = 1591.84 \text{ m}$$

(d) What is the total time that it takes to reach maximum altitude?

Time taken  $t'$  from  $1200 \text{ m}$  height to the maximum altitude (velocity is zero) is given by The first equation of motion as  $[v = u + a*t']$

Or  $0 = 87.64 + (-9.8)*t'$

[motion under gravity]

Or  $t' = 87.64/9.8 = 8.94 \text{ s}$ .

Hence the total time taken to reach the maximum altitude will be

$$t + t' = 27.39 + 8.94 = 36.33 \text{ s.}$$

(e) With what velocity does the rocket strike the earth?

Consider the motion from the maximum altitude with downward direction positive, the initial velocity is zero, the displacement  $s = 1238.27 \text{ m}$ , and the acceleration is  $9.8 \text{ m/s}^2$ , hence the final velocity at the surface of the earth  $v$  is given by

$$v^2 = u^2 + 2*a*s$$

or  $v^2 = 0 + 2*9.8*1591.84 = 31200$

or  $v = 176.64 \text{ m/s.}$

(f) What is the total time that it is in the air?

Time to reach ground from the maximum altitude  $t''$  is given by

$$[s = ut + \frac{1}{2} at^2]$$

$$1591.84 = 0 + 0.5*9.8*t''^2$$

Gives  $t'' = 18.02 \text{ s}$

Hence the total time in air will be

$$t + t' + t'' = 27.39 + 8.94 + 18.02 = 54.35 \text{ s.}$$