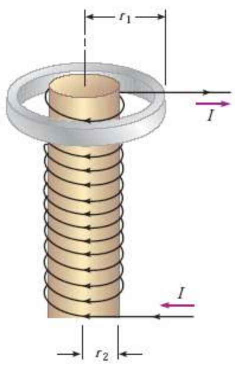
Q- An aluminum ring of radius $r_1$ and resistance $R$ is placed around the top of a long air-core solenoid with $n$ turns per meter and smaller radius $r_2$ as shown in Figure. Assume that the axial component of the field produced by the solenoid over the area of the end of the solenoid is one-half as strong as at the solenoid's center. Assume that the solenoid produces negligible field outside its cross-sectional area. The current in the solenoid is increasing at a rate of dI/dt.

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(a) What is the induced current in the ring?

Field at the center of a long solenoid with current I and number of turns per unit length is given by

$$B_C = \mu_0 n I$$

Field at the end of the solenoid

$$B = \frac{1}{2}\mu_0 n I$$

Magnetic flux through the solenoid

$$\phi = BA = \frac{1}{2}\mu_0 n I * \pi r_2^2$$

As there in no field outside the solenoid, the flux through the ring will be the same and hence the induced EMF will be given according to Faraday's law as

$$e = -\frac{d\phi}{dt} = -\frac{1}{2}\mu_0 n * \pi r_2^2 * \frac{dI}{dt}$$

And the current will be

$$i = \frac{e}{R} = -\frac{1}{2R}\mu_0 n * \pi r_2^2 * \frac{dI}{dt}$$

(b) At the center of the ring, what is the magnetic field produced by the induced current in the ring?

The field at the center of a current carrying ring is

$$B = \frac{\mu_0 I}{2r}$$

Hence the current at the center of solenoid due to the current in the ring will be

$$B_{ring} = \frac{\mu_0}{2r_1} * \frac{1}{2R}\ \mu_0 n\pi r_2^2 * \frac{dI}{dt}$$

Or $B_{ring} = \frac{\mu_0^2 n\pi r_2^2}{4Rr_1} * \frac{dI}{dt}$