Q- A snowmobile weighing 1000 lbs starts from rest, coasts down a 5^0 incline. If the force of friction opposing the motion is 40 lbs and the air resistance in pounds is numerically equal to 1.5 times the speed in feet per second, find:
a) An expression of the acceleration
b) An expression for the speed as a function of time
c) Speed after 10 seconds
d) The terminal speed of the snowmobile.

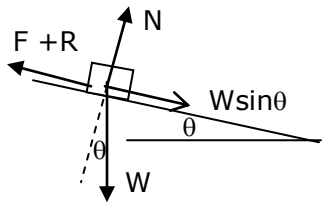
First of all the units

Pound (lb) is the unit of weight (gravitational force) in FPS system of units.
Unit of mass in FPS is slug, which is the mass of a body whose weight is
g = 32.17 f/s^2

Mass of the snowmobile M = 1000 /32.17 = 31.08 slug

(a) Let the mass of the snowmobile is M and the force of friction is F. If at time t its velocity is v then the air resistance will be R = 1.5*v. The forces acting on the snowmobile along the slope are the component of its weight (W = Mg) in the direction on motion, friction F and the air resistance R in the direction opposite to its motion. Hence the equation of motion for the snowmobile is given by

W*sin θ – F – R = Ma where a is the acceleration of the snowmobile.



Hence the acceleration of the snowmobile is given by

$$a = \left(\frac{W\sin\theta - F - R}{M}\right)$$

or $$a = \frac{1000*\sin 5^0 - 40 - 1.5*v}{31.08}$$

or a = (47.16 – 1.5*v)/31.08

or a = 1.52 – 0.048*v

(b) Using the equation above we can write the acceleration as

a = dv/dt = 1.52 – 0.048*v

This is a variable separable differential equation and hence can be written as

$$\frac{dv}{1.52 - 0.048v} = dt$$

Integrating we get

$$\int_0^v \frac{dv}{1.52 - 0.048v} = \int_0^t dt$$

Or $$\frac{1}{-0.048}\left[\ln(1.52 - 0.048v)\right]_0^v = t$$

Or $$\ln\left(1-\frac{0.048*v}{1.52}\right)=-0.048*t$$

Or $$v=\frac{1.52}{0.048}\left(1-e^{-0.048*t}\right)=31.67*\left(1-e^{-0.048*t}\right)$$

This is the expression for velocity as a function of time.

(c) The velocity at t = 10 s is obtained by substituting this time in above equation as

$$v_{10}=31.67*\left(1-e^{-0.048*10}\right)=31.67*\left(1-e^{-0.48}\right)=31.67*(1-0.62)$$

or $$v_{10}=31.67*0.38=12.03 \text{ f/s}$$

(d) As the velocity increases air resistance increases with it hence net force and acceleration decreases. After infinitely long time the net force is limiting to zero and after that body starts moving with constant velocity. This velocity is called terminal velocity and is given by substituting t = infinite in to the equation, thus

$$v_T=31.67*\left(1-e^{-\infty}\right)=31.67*(1-0)=31.67 \text{ f/s}$$