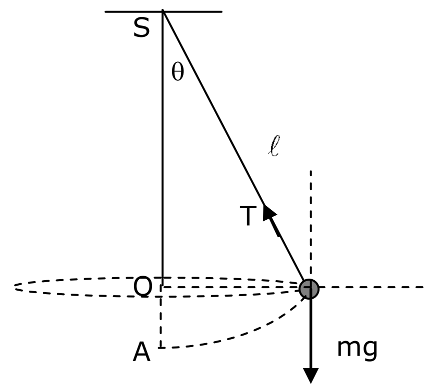
Q- A small bob of mass m is suspended by a string of length *l*. the bob is moving in a horizontal circle with constant speed, such that the string makes an angle θ with the vertical. Find:
a) Its speed
b) Angular momentum and
c) work done to bring it to rest.



(a) Let the mass is circulating with the velocity v on the circular path. The radius of the path r will be given in terms of the angle θ and length *l* of the string as

$r = \ell * \sin\theta$

Now the forces acting on the mass are

1. the weight of the mass mg vertically downwards and
2. The tension in the string T.

The tension in the string can be resolved in horizontal and vertical components and the components are T*sinθ and T cosθ respectively.

The vertical component of the tension balances the weight mg of the mass as there in no motion in vertical direction and hence

$T*\cos\theta - mg = 0$

Or $T*\cos\theta = mg$ ---------- (1)

The horizontal component of the tension T will act as centripetal force acting on the mass to make in move on the circular path with constant speed and is given by

$T*\sin\theta = mv^2/r$ ---------- (2)

Dividing equation 2 by 1 we get

$$\tan\theta = \frac{v^2}{g*r}$$

Or $v = \sqrt{g*r*\tan\theta} = \sqrt{g*l\sin\theta*\tan\theta}$ {substituting the value of r}

(b)

The angular momentum of a particle is also defined as the moment of momentum about the axis of rotation and is measured as the product of the momentum and the distance of the particle from the axis of rotation. Hence angular momentum L of the mass about the vertical axis of rotation from the center of the circle is

$L = m*v*r = m*\sqrt{g*l\sin\theta*\tan\theta}*l\sin\theta$

Or $L = ml\sin\theta\sqrt{g*l\sin\theta*\tan\theta} = m\sqrt{g\tan\theta}*(l\sin\theta)^{3/2}$

(c)

The work required to bring the mass to the rest.

Actually the mass possesses the kinetic energy and potential energy with respect to the lowest equilibrium position and to come to the rest it must lose this energy. Means that the body can do work before coming to rest hence the work done on the body will be negative. According to work energy rule we have.

Work done on the body = increase in energy = final energy – initial energy

$$W = 0 - (\text{initial P.E.} + \text{initial K.E.})$$

Or $$W = 0 - (m*g*h + \tfrac{1}{2} mv^2)$$

Now the height of the mass from the equilibrium position will be

$$h = SA - SO = \ell - \ell \cos\theta = \ell(1 - \cos\theta)$$

Hence $$W = -[mg * \ell (1 - \cos\theta) + \tfrac{1}{2} mg * \ell \sin\theta * \tan\theta]$$

Negative sign shows that the work is done by the bob on the stopping agency.