

Q- During a 2.5 hour run the metabolism of a marathon runner generates excessive body heat at an average rate of 4200 KJ per hour. Assume that the body (surface area 2 m²) loses heat at a rate of 50 W by convection.

a) Determine the rate at which radiation dissipates heat. Assume a skin temperature of 32 degrees C and an air temperature of 25 degrees C. The emissivity of the athlete's skin is 0.80.

b) Determine the mass of sweat that must evaporate per hour to keep the body temperature of the athlete constant during the race.

a)

The radiant power is given by Stephan's law as

$$\frac{dQ}{dt} = \sigma \epsilon A (T^4 - T_0^4)$$

Where σ is Stephan's constant, ϵ is the emissivity, A is the area of the body, T is absolute temperature and T_0 is the temperature of the surrounding. Hence

$$\frac{dQ}{dt} = 5.67 * 10^{-8} * 0.80 * 2.0 * (305^4 - 298^4) = 69.6 \text{ W}$$

b)

Let the mass of the sweat evaporate per hour be M then the heat loss will be ML where L (=2256 KJ/ Kg) is the latent heat of evaporation given by

$$Q = M * 2256 \text{ KJ}$$

Heat loss per hour due to convection = 50*3600 J = 180 KJ

Heat loss due to radiation = 69.6*3600 J = 250.56 KJ

Now to keep the temperature constant the total heat produced in an hour must be equal to the heat lost in the three ways hence for one hour =(3600 s) Hence

$$M * 2256 \text{ KJ} + 180 \text{ KJ} + 250.56 \text{ KJ} = 4200 \text{ KJ}$$

Or
$$M = \frac{4200 - 180 - 250.56}{2256} = 1.67 \text{ kg}$$
