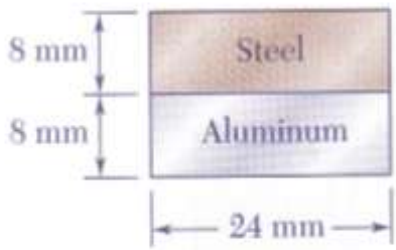
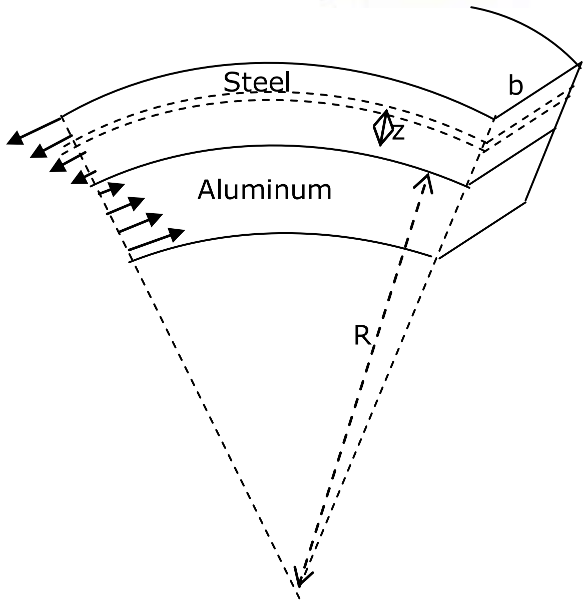
Q- A steel bar (Y_{steel} = 210 GPa) and an aluminum bar ($Y_{aluminum}$) = 10 GPa are bonded together to form the composite bar shown. Determine the maximum stress in (a) the aluminum, (b) the steel, when the bar is bent towards aluminum bar about a horizontal axis, with a torque M = 60 Nm.



Let the composite bar is bend as in figure. Due to symmetry we can see that the steel bar is get elongated and the aluminum bar get compressed. The inter face will remain at the same length.

Consider a small part of the bar, subtending angle θ at the center of curvature. The radius of the interface MN is R. The length of every layer when there is no bending will be = Rθ.



When the rod is bent, consider a thin layer of thickness dz of the rod at distance z from the interface MN.

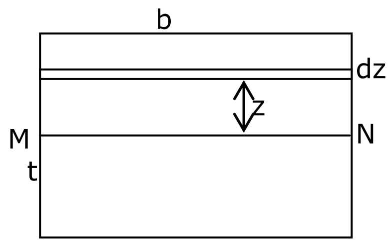
Radius of this layer is R+z and hence the new length of the layer will be (R+z)*θ

So due to bending increase in the length of the layer is (R+z)*θ – Rθ = zθ.

Thus strain in this layer = z θ/ R θ = z/R.

If the bending in the bar is small the shearing or the change in volume are negligible thus only longitudinal strain is considered and hence the stress in this layer is given by

Stress = Y*strain = Y*z/R ---------- (1)



And the force on this layer = area*stress
= (b*dz)*(Y*z/R)

And the moment of this force (torque) about the neutral layer MN will be

dM = (b*dz)*(Y*z/R)*z = (Y*b/R) z^2dz

Hence bending moments produced in the steel bar is given by integrating for the thickness 't' of the steel bar as

$$M_{steel} = \frac{Y_{steel}b}{R}\int_0^t z^2\,dz = \frac{Y_{steel}bt^3}{3R}$$

Or $$M_{steel} = \frac{(210*10^9\ Pa)(24*10^{-3}\ m)\ (8*10^{-3}\ m)^3}{3R} = \frac{860.2}{R}\ Nm$$

Similarly the bending moment for the aluminum bar will be

$$M_{Aluminum} = \frac{Y_{Aluminum}b}{R}\int_0^t z^2\,dz = \frac{Y_{Aluminum}bt^3}{3R}$$

Or $$M_{Aluminum} = \frac{(10*10^9\ Pa)(24*10^{-3}\ m)\ (8*10^{-3}\ m)^3}{3R} = \frac{40.96}{R}\ Nm$$

Now as the both moments about MN are in the same direction (due to elongation and compression) total bending moment will be

M = (860.2 + 40.96)/R = 901.16/R Nm

Now as the bar is in equilibrium, this bending moment is balanced by applied moment and hence

901.16/R = 60

Or R = 15.02 m

Now from equation (1) Stress in any layer is Y*z/R it is proportional to z, the distance of the layer form the neutral layer MN and hence is maximum for the two boundary layers is for z = t (the thickness)

Hence

Maximum stress in steel bar = $\frac{Y_{steel}\ \mathrm{t}}{\mathrm{R}} = \frac{210*10^9*8*10^{-3}}{15.02} = 1.12 * 10^8\ Pa$

And

Maximum stress in aluminum bar = $\frac{Y_{Aluminum}\ \mathrm{t}}{\mathrm{R}} = \frac{10*10^9*8*10^{-3}}{15.02} = 5.33 * 10^6\ Pa$