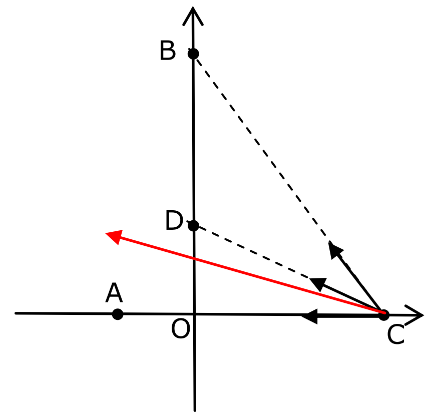
Q- Four identical point particles (each having mass m=5000kg) are located at the points (-2,0), (0,8), (6,0) and (0,3) in the xy plane. All positions are measured in meters. Determine the magnitude of the total gravitational force on the particle located at (6,0) due to the other three particles.



The gravitational force of attraction between two particles of mass m_1 and m_2 at distance r from each is given by Newton's law of universal gravitation as

$$F = \frac{Gm_1 m_2}{r^2}$$

The direction of this force is along the line joining the two masses and towards each other.

Here the mass of all particles is same m; hence the magnitude of force between any two is given by

$$F = \frac{Gm^2}{r^2}$$

Here r is the distance between the two particles.

The particle at point C (6.0) is experiencing force of gravitation due to other three and the net force is the resultant of the three forces. The best way is to write the forces in component form.

i) The force due to the particle at A

The distance between the particles is 6 – (-2) = 8 m

Hence magnitude of the force will be

$$F_A = \frac{Gm^2}{r^2} = \frac{6.67*10^{-11}*5000^2}{8^2} = 2.6*10^{-5} \text{ N}$$

As this force is towards negative x direction, given by

$$\vec{F}_A = -2.6*10^{-5}\hat{i}$$

Here $\hat{i}$ is unit vector in the x direction.

ii) The force due to the particle at B

The distance between the particles is $\sqrt{6^2+8^2} = 10$ m

Hence magnitude of the force will be

$$F_B = \frac{Gm^2}{r^2} = \frac{6.67*10^{-11}*5000^2}{10^2} = 1.67*10^{-5} \text{ N}$$

Components of this force in x and y directions is given by

$$F_{Bx} = F_B \cos(BCO) = F_B*(OC/BC) = 1.67*10^{-5}*(6/10) = 1.00*10^{-5} \text{ N}$$

And F_{By} = F_B sin (BCO) = F_B*(OB/BC) = 1.67*10^{-5}*(8/10) = 1.33*10^{-5} N

Hence the force F_B can be written as

$$\vec{F}_B = -1.00*10^{-5}\hat{i} + 1.33*10^{-5}\hat{j}$$

Where $\hat{j}$ is unit vector in y direction.

iii) The force due to the particle at D

The distance between the particles is $\sqrt{6^2+3^2} = 6.71$m

Hence magnitude of the force will be

$$F_D = \frac{Gm^2}{r^2} = \frac{6.67*10^{-11}*5000^2}{6.71^2} = 3.70*10^{-5}\text{ N}$$

Components of this force in x and y directions is given by

F_{Dx} = F_D cos (DCO) = F_D*(OC/DC) = 3.70*10^{-5}*(6/6.71) = 3.31*10^{-5} N

And F_{Dy} = F_D sin (DCO) = F_D*(OD/DC) = 3.70*10^{-5}*(3/6.71) = 1.65*10^{-5} N

Hence the force F_D can be written as

$$\vec{F}_D = -3.31*10^{-5}\hat{i} + 1.65*10^{-5}\hat{j}$$

Hence the resultant of the three forces is given by

$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_D$$

Or $\vec{F} = (-2.6 - 1.00 - 3.31)*10^{-5}\hat{i} + (0 + 1.33 + 1.65)*10^{-5}\hat{j}$

Or $\vec{F} = (-6.91)*10^{-5}\hat{i} + (2.98)*10^{-5}\hat{j}$

Magnitude of this resultant will be

$$F = \sqrt{(-6.91)^2 + 2.98^2}*10^{-5} = 7.53*10^{-5}\text{ N}$$

And its direction is given by

Tanθ = 2.98/(-6.91) = - 0.43123

Of θ = - 23.33 deg

Means that the total force on C is 7.53*10^{-5} N and is in the direction making an angle of 23.33 deg with the negative x axis.