Q- At what altitude above the Earth's surface would your weight be three-fourths of what it is at the Earth's surface? Assume r_e = 6.371 ×10^3 km.

Weight of a body is the force by which a body is attracted towards the Earth. This force of attraction is due to the gravitational field of earth. The force of attraction is given by the Newton's law of universal gravitation and its magnitude is mathematically written as

$$F = \frac{GM_e m}{r^2}$$

Here M_e is the mass of earth, m is the mass of the body and r is the distance of the body from the center of the earth. G is the constant known as the universal gravitation constant.

Due to this force the acceleration produced in the body is called acceleration due to gravitation denoted by g and its value at the surface of earth is 9.8 m/s^2. Hence the weight of the body of mass m at any place is written ad mg, which is the force of attraction. As we move away from the surface of earth the distance r increases and hence the weight of the body decreases.

Let the mass of the body is m. The weight of the body at the surface of earth will be given by

$$W = \frac{GM_e m}{r_e^2} \qquad \text{---------- (1)}$$

Let at the height h from the surface of the earth the weight of the body is ¾ W, then the distance of the body from the center of earth will be r_e + h and hence its weight is given by

The weight of the body at this altitude is given by

$$\frac{3W}{4} = \frac{GM_e m}{\left(r_e + h\right)^2} \qquad \text{---------- (2)}$$

Dividing equation 1 by equation 2 we get

$$\frac{4}{3} = \frac{\left(r_e + h\right)^2}{r_e^{\,2}}$$

Or $$\frac{4}{3} = \left(1 + \frac{h}{r_e}\right)^2$$

Gives $$\frac{2}{\sqrt{3}} - 1 = \frac{h}{r_e}$$

Or $$h = \left(\frac{2}{\sqrt{3}} - 1\right) r_e = 0.1547 * r_e = 0.1547 * 6.371 * 10^3 = 985.6 \text{ km}$$

Hence the altitude will be 985.6 km.