Q- A monochromatic light source emits 100w of electromagnetic power uniformly in all directions.
a) Calculate the average electric- field energy density 1.00 m from the source.

As the energy emitted is distributed uniformly in all directions, intensity (energy incident per unit time per unit area) is spherically symmetric. Hence the power emitted by the sours P and the intensity at a distance r from the source is related by the formula

$$I = \frac{P}{4\pi r^2}$$

Now as the intensity of the electromagnetic wave at a point is given by

$$I = \frac{1}{\mu_0 c} E_{rms}^2$$

We get

$$I = \frac{1}{\mu_0 c} E_{rms}^2 = \frac{P}{4\pi r^2}$$

Gives $E_{rms}^2 = \dfrac{\mu_0 c P}{4\pi r^2}$ ------------------- (1)

The energy density in an electric field is given by

$$\frac{1}{2} \in_0 E^2$$

And hence the average electric field energy density at this point will be given by substituting from equation 1 as

$$\rho_E = \frac{1}{2} \in_0 E_{rms}{}^2 = \frac{\in_0 \mu_0 c P}{2 * 4\pi r^2} = \frac{P}{2 * 4\pi r^2 * c}$$ [As $\in_0 \mu_0 = 1/c^2$]

Gives

$$\rho_E = \frac{100}{2*4*3.1416*1.00^2*3*10^8} = 1.33*10^{-8} \text{ J/m}^3$$

b) Calculate the average magnetic-field energy density at the same distance from the source.

In electromagnetic wave the magnetic field is related to electric field as E = c*B so we have

$$B_{rms}^2 = \frac{E_{rms}^2}{c^2} = \frac{\mu_0 P}{4\pi r^2 * c}$$ [using equation 1]

And the average magnetic field energy density at the same distance will be

$$\rho_B = \frac{B_{rms}^2}{2\mu_0} = \frac{P}{2*4\pi r^2 * c} = \frac{100}{2*4*3.1416*1.00^2*3*10^8} = 1.33*10^{-8} \text{ J/m}^3$$

[Both average energy densities are same as the fields are develops due to change in each other]