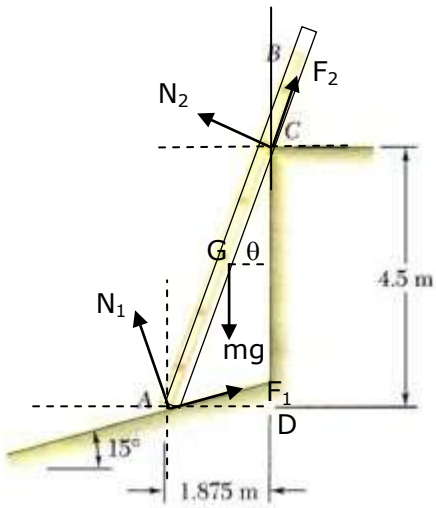
Q- A 6.5 m ladder AB of mass 10 kg leans against a wall as shown. Determine the smallest value of coefficient of static friction between the surfaces (considering same everywhere) for which equilibrium can be maintained.



Let the value of coefficient of static friction μ_s is just sufficient and hence the ladder is in limiting equilibrium. The frictional force at the two contact points will be limiting friction and is given by the product of coefficient of static friction and the normal reaction.

For equilibrium of a body, the net force acting on the body must be zero and the net torque about any point (axis of rotation) must be zero.

The angle of inclination of the ladder with the horizontal is given by solving the triangle ACD as

$\tan\theta = CD/AD = 4.5/1.875 = 2.4$

Gives $\theta = 67.38^o$

and $AC = \sqrt{AD^2 + DC^2} = \sqrt{1.875^2 + 4.5^2} = 4.875m$

The floor is having inclination to the horizontal

$\alpha = 15^o$

Now the forces acting on the ladder are

The weight mg of the ladder, the normal reaction of the floor N_1 at A, limiting friction force F_1 along the floor at A (equal to $\mu_s N_1$), normal reaction of the upper corner N_2 at C, and limiting friction force F_2 at C (equal to $\mu_s N_2$).

As the ladder is in equilibrium the net force and hence the components of all forces in horizontal direction and vertical direction are separately zero.

Resolving the Forces in horizontal direction and adding them we have

$F_1 * \cos\alpha - N_1 * \sin\alpha + F_2 * \cos\theta - N_2 \sin\theta = 0$

Or $\mu_s N_1 * \cos\alpha - N_1 * \sin\alpha = N_2 \sin\theta - \mu_s N_2 * \cos\theta$

Or $$\frac{N_1}{N_2} = \frac{\sin\theta - \mu_s \cos\theta}{\mu_s \cos\alpha - \sin\alpha} \qquad \text{------- (1)}$$

Resolving the Forces in vertical direction and adding them we have

$F_1 * \sin\alpha + N_1 * \cos\alpha + F_2 * \sin\theta + N_2 \cos\theta - mg = 0$

Or $\mu_s N_1 * \sin\alpha + N_1 * \cos\alpha + \mu_s N_2 * \sin\theta + N_2 \cos\theta - mg = 0$

Or $N_1 (\mu_s * \sin\alpha + \cos\alpha) + N_2 (\mu_s \sin\theta + \cos\theta) = mg$ ------- (2)

Substituting value of N_1 in equation 2 from equation 1 we have

$$N_2\left(\frac{\sin\theta-\mu_s\cos\theta}{\mu_s\cos\alpha-\sin\alpha}\right)\left(\mu_s\sin\alpha+\cos\alpha\right)+N_2\left(\mu_s\sin\theta+\cos\theta\right)=mg$$

Or $$N_2\frac{\left(\sin\theta-\mu_s\cos\theta\right)\left(\mu_s\sin\alpha+\cos\alpha\right)+\left(\mu_s\cos\alpha-\sin\alpha\right)\left(\mu_s\sin\theta+\cos\theta\right)}{\mu_s\cos\alpha-\sin\alpha}=mg$$

Or $$N_2\frac{\left(\mu_s^{\,2}+1\right)\sin\left(\theta-\alpha\right)}{\mu_s\cos\alpha-\sin\alpha}=mg \qquad \text{------- (3)}$$

Now for equilibrium the torque about any point should be zero hence taking torque of all forces about point A we have

(Torque is the product of force and perpendicular distance of its line of action from the point A and hence torque due to N_1, F_1 and F_2 are zero)

Thus N_2*AC – mg*AG cosθ = 0

Gives N_2 = mg*AG cosθ /AC = mg*3.25cosθ/4.875 (G is the center of ladder)

Or N_2 = 0.2564 mg

Substituting the value of N in equation 3 we have

$$0.2564mg*\frac{\left(\mu_s^{\,2}+1\right)\sin\left(\theta-\alpha\right)}{\mu_s\cos\alpha-\sin\alpha}=mg$$

Or $$\frac{\left(\mu_s^{\,2}+1\right)\sin\left(\theta-\alpha\right)}{\mu_s\cos\alpha-\sin\alpha}=3.9$$

Substituting values of θ and α in above equation we have

$$\left(\mu_s^{\,2}+1\right)\sin\left(67.38-15\right)^0=3.9\left(\mu_s\cos15^0-\sin15^0\right)$$
$$\left(\mu_s^{\,2}+1\right)0.79=3.9\left(0.966\mu_s-0.259\right)$$

Or $$0.79\mu_s^2-3.77\mu_s+1.8=0$$

Or $$\mu_s=\frac{3.77\pm\sqrt{3.77^2-4*0.79*1.8}}{2*0.79}=\frac{3.77\pm\sqrt{8.525}}{2*0.79}=\frac{3.77\pm2.92}{1.58}$$

Or μ_s = 0.54.

(Coefficient of friction can not be more then 1)