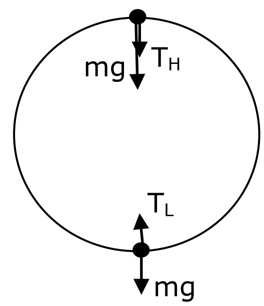
Q- A ball is tied to the end of an ideal small spring and when it hangs freely at rest, it is found to stretch the spring by 8cm. The ball is now attached to a long string and whirled around in a vertical circle at a constant angular speed. Determine the difference between the maximum amount that the spring will stretch and the minimum amount that the spring will stretch.

The force constant of a spring is the force required per unit extension of the spring and hence the force constant of the spring is given by

$$K = mg/\Delta\ell$$

As the radius of the vertical circle is not given let it be R and constant. ($R >> \Delta\ell$, so that change in radius due to the stretch in the spring is negligible)

As the ball is moving on a circular path it requires centripetal force which is given by the resultant of the tension in the string (spring) and the weight of the ball. The angular speed of the ball is constant hence the centripetal force F will be constant.



At the lowest position

$$T_L - mg = F$$

Or $$T_L = F + mg$$

And at the highest position

$$mg + T_H = F$$

or $$T_H = F - mg$$

This shows that the maximum value of the tension in the spring will be at lowest point and the minimum is at highest point. The difference in the tensions is given by

$$T_L - T_H = 2\ mg$$

The extensions in the spring is given by

$$\Delta\ell_{max} = \frac{T_L}{K}$$

And $$\Delta\ell_{min} = \frac{T_H}{K}$$

Hence

$$\Delta\ell_{max} - \Delta\ell_{min} = \frac{T_L}{K} - \frac{T_H}{K} = \frac{T_L - T_H}{K} = \frac{2mg}{K}$$

Substituting the value of K we have

$$\Delta\ell_{max} - \Delta\ell_{min} = \frac{2mg}{K} = 2\Delta\ell = 2*8 = 16\ \text{cm}.$$

(It is very difficult to make the ball move in a vertical circle with constant angular speed. It is assumed here that some additional variable torque is moving it like that.)