Q- Particles of masses m_1 = 3kg and m_2 = 5kg move on a collision course with velocities: u_1 = (7 i + 3 j) m/s and u_2 = (3 i + 7 j) m/s, respectively. After they collide, their velocities are v_1 = (a i + b j) m/s and v_2 = (c i + 4 j) m/s, respectively. If the collision is perfectly elastic, determine all possible values of a, b and c.

As there is no external force acting on the system of the two particles, the momentum of the system will remain conserved.

Conserving momentum in x direction we have

$$m_1u_{1x} + m_2v_{2x} = m_1v_{1x} + m_2v_{2x}$$

Or 3*7 + 5*3 = 3*a + 5*c
Gives 3a + 5c = 36 ---------- (1)

Similarly conserving momentum along y direction we have

$$m_1u_{1y} + m_2u_{2y} = m_1v_{1y} + m_2v_{2y}$$

Or 3*3 + 5*7 = 3*b + 5*4
Gives b = 8 ---------- (2)

And as the collision is perfectly elastic collision we can conserve kinetic energy before and after collision and hence

$$\frac{1}{2}m_1\left(u_{1x}^2+u_{1y}^2\right)+\frac{1}{2}m_2\left(u_{2x}^2+u_{2y}^2\right)=\frac{1}{2}m_1\left(v_{1x}^2+v_{1y}^2\right)+\frac{1}{2}m_2\left(v_{2x}^2+v_{2y}^2\right)$$

Or $3\left(7^2+3^2\right)+5\left(3^2+7^2\right)=3\left(a^2+b^2\right)+5\left(c^2+4^2\right)$

Or $464=3\left(a^2+8^2\right)+5\left(c^2+4^2\right)$

Or $464=3a^2+192+5c^2+80$

Or $3a^2+5c^2=192$

Gives $3a^2+5\left(\frac{36-3a}{5}\right)^2=192$ [substituting the value of c from equation 1]

Or $15a^2+\left(36-3a\right)^2=960$

Or $24a^2-216a+336=0$

Or $a^2-9a+14=0$

Gives a = 2 and a = 7

And c = $\pm$6 and c = $\pm$3

Hence the value sets of a, b and c are (2, 8, $\pm$6) and (7, 8, $\pm$3)