Q (a) How much work is required to accelerate a proton from rest to a speed of 0.998 c. (b) What would be the momentum of this proton.

(a) The work done on the proton will increase its kinetic energy

The increase in the kinetic energy of a particle when it is accelerated form rest to velocity v is given by

ΔE = total energy – rest energy

Or $$\Delta E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 c^2$$

Or $$\Delta E = \frac{m_0 c^2}{\sqrt{1-\frac{0.998^2 c^2}{c^2}}} - m_0 c^2$$

Or $$\Delta E = \frac{m_0 c^2}{\sqrt{1-0.998^2}} - m_0 c^2$$

Or $$\Delta E = 14.82\, m_0 c^2 = 14.82 * 1.67 * 10^{-27} * (3 * 10^8)^2$$

Or $$\Delta E = 2.23 * 10^{-9}\, J$$

Thus the work done is equal to the increase in its kinetic energy

Or $$W = 2.23 * 10^{-9}\, J$$

(b) The relativistic momentum of the proton is given by

$$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.67*10^{-27}*0.998*3*10^8}{\sqrt{1-0.998^2}} = 7.91 * 10^{-18}\, kg\, m/s$$