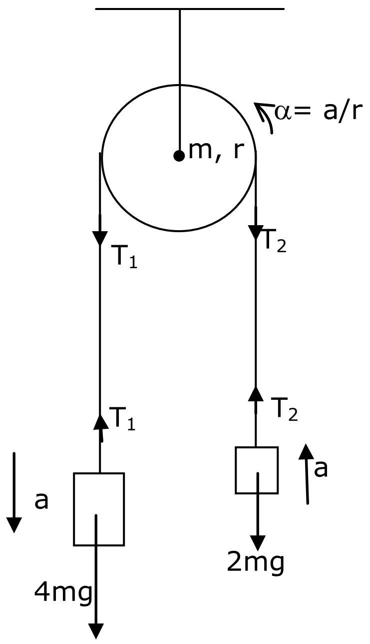
Q- A pulley wheel of radius r and mass m is free to rotate about a fixed smooth horizontal axis through its centre perpendicular to the plane of the pulley? A light inextensible string passes over the pulley and carries particles of mass 4m and 2m, one at each end of the string. It can be assumed that the pulley wheel can be modeled as a uniform circular disc and that the string does not slip on the pulley. Find the tension in each vertical part of the string.



The friction between the pulley and the string will create a torque on the pulley and it will have an angular acceleration say α.

This friction on the string will change the tension in it hence the tension on both sides of the pulley will be different say T_1 and T_2.

As the angular acceleration in the pulley is α and the string is inextensible the acceleration in the two masses will be $a = \alpha*r$ downward for 4m and upward for mass 2m.

Now the forces acting on 4m mass are its weight 4m (downward) and the tension in the string T_1 (upward) hence writing equation of motion [F = ma] for mass 4m with downward positive we have

$4mg - T_1 = 4ma$ ------------- (1)

And for mass 2m with upward positive

$T_2 - 2mg = 2ma$ ------------- (2)

Now torque on the pulley is given in anticlockwise direction by

$\tau = T_1 r - T_2 r$

Writing equation of rotational motion for the pulley we have

$\tau = I*\alpha$

Or $(T_1 - T_2)*r = I*\alpha$

Here I is the moment of inertia of the pulley (disk) given by $\frac{1}{2} mr^2$. Substituting the value in equation above we have

$(T_1 - T_2)*r = \frac{1}{2} mr^2*\alpha$

Or $T_1 - T_2 = \frac{1}{2} m r*\alpha$

Or $T_1 - T_2 = \frac{1}{2} ma$ [as $a = r*\alpha$] ------------- (3)

Adding all three equations 1, 2 and 3 we have

$4mg - 2mg = 4ma +2 ma + \frac{1}{2} ma$

Or $2mg = (13/2) ma$

Gives $ma = 4mg/13$.

Substituting this value in equation 1 we get

$4mg - T_1 = 4(4mg/13)$

Gives $T_1 = 36mg/13$

And substituting in equation 2 we get

$T_2 - 2mg = 2*(4mg/13)$

Or $T_2 = 2mg + 2(4mg/13) = 34mg/13$

Hence the tensions in the strings will be

$\mathbf{T_1 = 36mg/13}$

And $\mathbf{T_2 = 34mg/13}$