Q- White light (λ = 400 – 700 nm) is incident on a 600 line/mm diffraction grating. What is the width of the first order rainbow on the screen 2.0 m behind the grating?

The angle of deviation of the first order maximum is given by

$$\sin\theta = \frac{\lambda}{d}$$

Here d is the line spacing and λ is the wavelength of the light.

The distance x of this maximum from the central maximum on the screen is given by

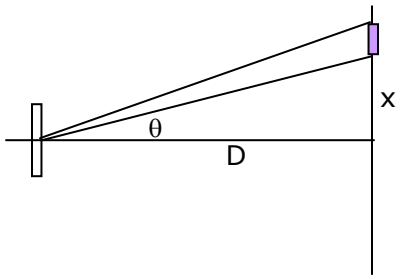
$$x/D = \tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{\lambda/d}{\sqrt{1-(\lambda/d)^2}} = \frac{\lambda}{\sqrt{d^2-\lambda^2}}$$

or $$x = D*\frac{\lambda}{\sqrt{d^2-\lambda^2}}$$

Now the grating spacing $d = 1*10^{-3}/600 = 1.6667*10^{-6}$ m = 1667 nm

Hence for the wavelength 400 nm the distance of the first order maximum is

$$X_1 = 2.0*\frac{400}{\sqrt{1667^2-400^2}} = 0.494\,\text{m}$$



And the same for 700 nm wave

$$X_2 = 2*\frac{700}{\sqrt{1667^2-700^2}} = 0.925\,\text{m}$$

Hence the width of the spectrum (rainbow)

$$W = x_2 - x_1 = 0.925 - 0.494 = 0.431\ \text{m}$$