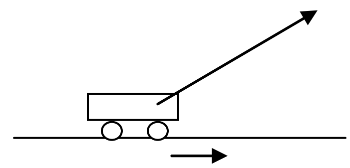
Q- In each of these, the $x-$ and $y-$components of a vector are given. Find the magnitude and direction of the vector.

Reading:

A vector has its effect in the direction other than its own direction too. A toy cart pulled by a child by a string making an angle to the horizontal pulls the cart in the horizontal direction means force applied at that angle has it effect in horizontal direction as well, due to which the cart moves horizontally.



The effect of the vector $\vec{A}$ in a direction making angle θ is called its projection and given by $A*\cos\theta$. Clearly the projection in perpendicular direction is zero. That is why we generally resolve (break up) a vector in two (or three) components perpendicular to each other and the components are the projections of the vectors in the corresponding directions.

Let the vector $\vec{A}$ is represented by two components Ax and Ay perpendicular to each other then we write it as

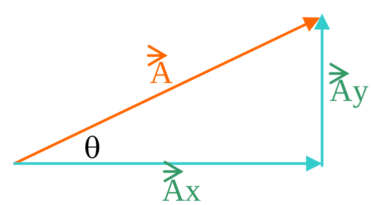
$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Where Ax and Ay are magnitudes of the components of the vector in x and y directions respectively given by

$A_x = A\cos\theta$ and

$A_y = A\cos(\pi/2 - \theta) = A\sin\theta$

$\hat{i}$ and $\hat{j}$ are the unit vectors in x and y directions respectively.



Hence the magnitude of vector A is given by

$$A = \sqrt{A_x^2 + A_y^2}$$

And the direction is given by

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

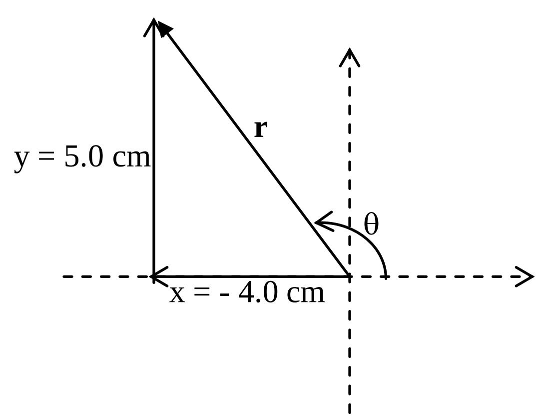
Solution:

(a) x = −4.0 cm, y = +5.0 cm.

Here $\vec{r} = x\hat{i} + y\hat{j} = 4.0\hat{i} + 5.0\hat{j}$

Hence $r = |\vec{r}| = \sqrt{(-4)^2 + (5)^2} = \sqrt{41} = 6.403$ cm

And the direction with x axis is given by



$$\theta = \tan^{-1}\left(\frac{5.0}{-4.0}\right) = \tan^{-1}(-1.25) = 180^0 - 51.34^0 = 128.66^0$$

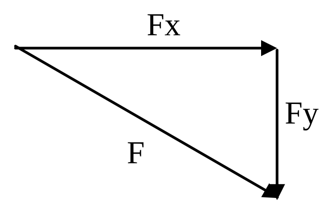
Hence the angle with y axis will be

$128.66 - 90 = 38.66^0$ CCW with +y axis

Direction: θ = 38.66 °CCW from the +y−axis.

(b) Fx = +160 N, Fy = −50.0 N.

$$\vec{F} = F_x\hat{i} + F_y\hat{j} = 160\hat{i} + (-50.0)\hat{j}$$



Hence the magnitude is given by

$$F = |F| = \sqrt{(160)^2 + (-50)^2} = 167.63\ N$$

And the direction will be given by

$$\theta = \tan^{-1}\left(\frac{-50.0}{160}\right) = \tan^{-1}(-0.3125) = -17.35^0$$ with + x axis

Direction: θ = 17.35 °CW from the +x−axis.