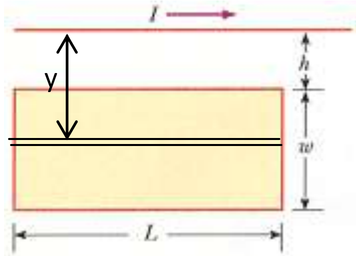
Q- A loop of wire in the shape of a rectangle of width *w* and length *L* and a long, straight wire carrying a current *I* lie on a tabletop as shown in Figure. Suppose the current is changing with time according to *I = a + bt,* where *a* and *b* are constants. Determine the emf that is induced in the loop if *b* = 19.0 A/s, *h* = 1.00 cm, *w* = 14.0 cm, and *L* = 135 cm.



The induced EMF in a loop is given by Faraday's law as

$$e = -\frac{d\phi_B}{dt}$$

Where ϕ_B is the magnetic flux through the loop

The magnetic field through the loop is not uniform and hence we have to find it with a way of integration. Consider a thin strip of thickness dy at a distance of y from the wire.

The area of the strip will be L*dy

Magnetic field at distance y from the wire carrying current I is given by

$$B = \frac{\mu_0 I}{2\pi y}$$

Hence the flux through the strip of length L and thickness dy will be

$$d\phi = B * dA = \frac{\mu_0 I}{2\pi y} * L * dy$$

So the flux through loop at the instant when current in the wire is I is given by

$$\phi = \int d\phi = \int_h^{w+h} \frac{\mu_0 I}{2\pi y} * L * dy = \frac{\mu_0 IL}{2\pi} \int_h^{w+h} \frac{1}{y} * dy = \frac{\mu_0 IL}{2\pi} \ln \frac{w+h}{h}$$

And the induced EMF in the loop is given by

$$e = -\frac{d\phi}{dt} = -\frac{\mu_0 L}{2\pi} \ln \frac{w+h}{h} * \frac{dI}{dt} \qquad \text{-------------------- (1)}$$

Now I = a + bt; Gives dI/dt = b

Substituting in equation 1 we get

$$e = -\frac{\mu_0 Lb}{2\pi} \ln \frac{w+h}{h} = -2*10^{-7}*1.35*19 \ln 15 = -1.389*10^{-5} volt$$

(Negative sign is showing the direction of induced emf as per Lenz law)