Q- A block with mass M attached to a horizontal spring with force constant k is moving with simple harmonic motion having amplitude A. At the instant when the block passes through its equilibrium position, a lump of putty with mass m is dropped vertically onto the block from a very small height and sticks to it. What fraction of the original mechanical energy is converted into heat?

The amplitude of the SHM is A and the force constant of the spring is k
The energy of a particle executing SHM is totally potential at the extreme positions and is given by

U_1 = PE = ½ KA^2

When the particle is at equilibrium its energy is totally kinetic and hence if the velocity of the particle at equilibrium position is v_M then it is given by using law of conservation of energy as

Gain in kinetic energy = loss in potential energy

Or ½ M v_M^2 = ½ k A^2

Or $v_M = \sqrt{\frac{k}{M}}A$ ------------------------------------- (1)

Now when the block is passing through equilibrium position the lump of putty is dropped over it means it was having zero velocity in horizontal direction.

As the putty sticks to the block and moves with the block after falling on it and as there is no external force on the system at that instant, applying law of conservation of linear momentum we can find the velocity v of the block with putty just after the putty is dropped as

Final momentum of the system = initial momentum of the system

Or (M + m)*v = M*v_M + m*0

Gives v = $\frac{M}{M+m}v_M = \frac{M}{M+m}\sqrt{\frac{k}{M}}A$ ------------------------------------- (2)

Hence the new total energy of the system of block and putty will be (totally KE at equilibrium position)

U_2 = ½ (M + m) v^2 = $\frac{M+m}{2}\left(\frac{M}{M+m}\right)^2\frac{k}{M}*A^2$ Using (2)

Or U_2 = $\frac{1}{2}\left(\frac{M}{M+m}\right)*kA^2$

Hence the fraction of the mechanical energy converted to heat will be

f = Less in mechanical energy/ original energy

or $f = \frac{U_1 - U_2}{U_1}$ = 1 - $\frac{U_2}{U_1} = 1$ - $\frac{\frac{1}{2}\left(\frac{M}{M+m}\right)*kA^2}{\frac{1}{2}kA^2}$

or $f = 1 - \frac{M}{M+m} = \frac{m}{M+m}$

or $f = \frac{m}{M+m}$